

Kinematics Model of Solids Conveying of LDPE with a Grooved Barrel

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Solids conveying in an extruder with a grooved barrel is difficult to analyze and calculate. The reason is that the barrel friction factor is not easily defined and approximated when grooves are present.

The kinematics model has been shown to accurately predict the solids conveying flow for smooth barrels without the use of friction factors. Here, the same approach is used to predict the solids conveying flow of an extruder with a grooved barrel.

KINEMATICS MODEL

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First, a review of kinematics model equation for solids conveying flow in an extruder. This is based on two previous works given at ANTEC 2003 and ANTEC 2004.

Kinematics model for solids conveying is based on the

- 1) conservation of mass,
and the**
- 2) solids conveying angle.**

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The kinematics model is based on the conservation of mass, which is formulated in cylindrical coordinates.

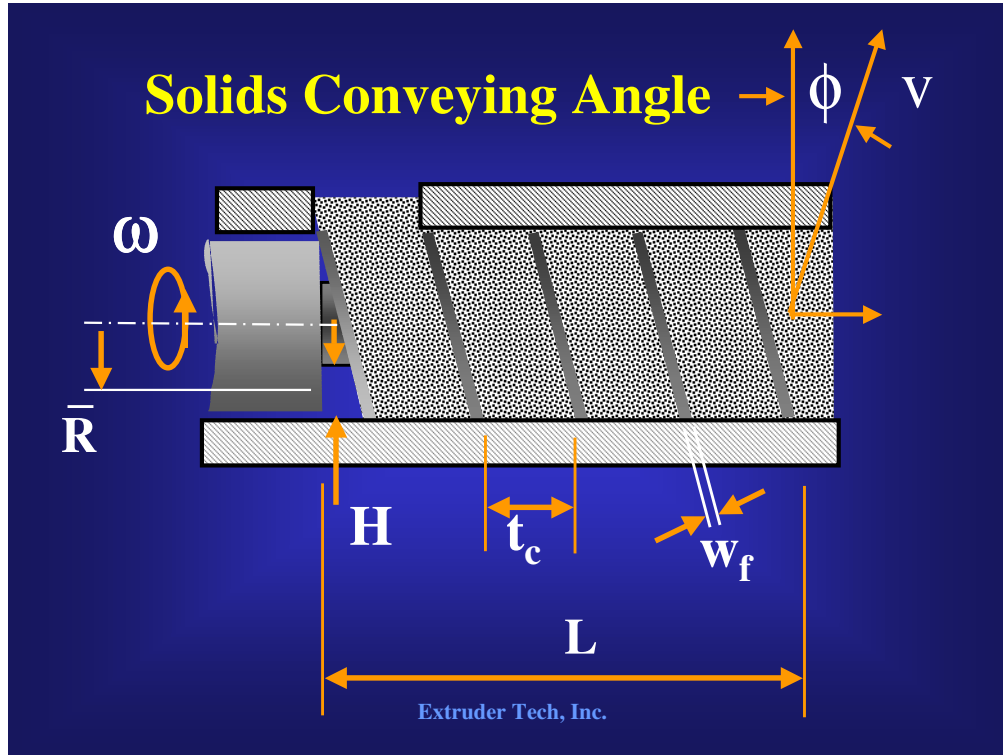
The solids conveying angle is a constitutive factor that replaces the use of friction factors.

Solids Conveying Angle

**DEFINED BY THE
KINEMATICS OF
THE SOLID BED**

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The solids conveying angle is defined by the forward angle of motion of the solids bed at the end of the solids conveying.



The geometric factors that are needed to describe the kinematics model with the solids conveying angle are shown. The bulk density of the polymer is also needed. It is best if the bulk density is known as a function of pressure. Bulk density can be found in the literature or from polymer suppliers.

Dimensionless Flow Rate

$$N_Q = \dot{m} / (\rho_B \omega \bar{R} H t_c)$$

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A dimensionless flow rate results from the model.

Kinematics Model: Flow Rate Equation

$$N_Q = (\tan \phi) / (\tan \theta + \tan \phi)$$

ϕ = Solids Conveying Angle

θ = Screw Helix Angle

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This is the basic solids conveying flow equation as predicted by the kinematics model. Again, cylindrical coordinates are used to derive this relationship.

Kinematics of Solids Flow versus Pressure

$$N_Q = \frac{\tan(\phi(p))}{(\tan \theta + \tan(\phi(p)))}$$

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The model is now used to predict the flow as a function of pressure by making the solids conveying angle a function of pressure. Data are used to evaluate the solids conveying angle as a function of pressure.

SOLIDS CONVEYING ANGLE vs PRESSURE

$$\phi(p)$$

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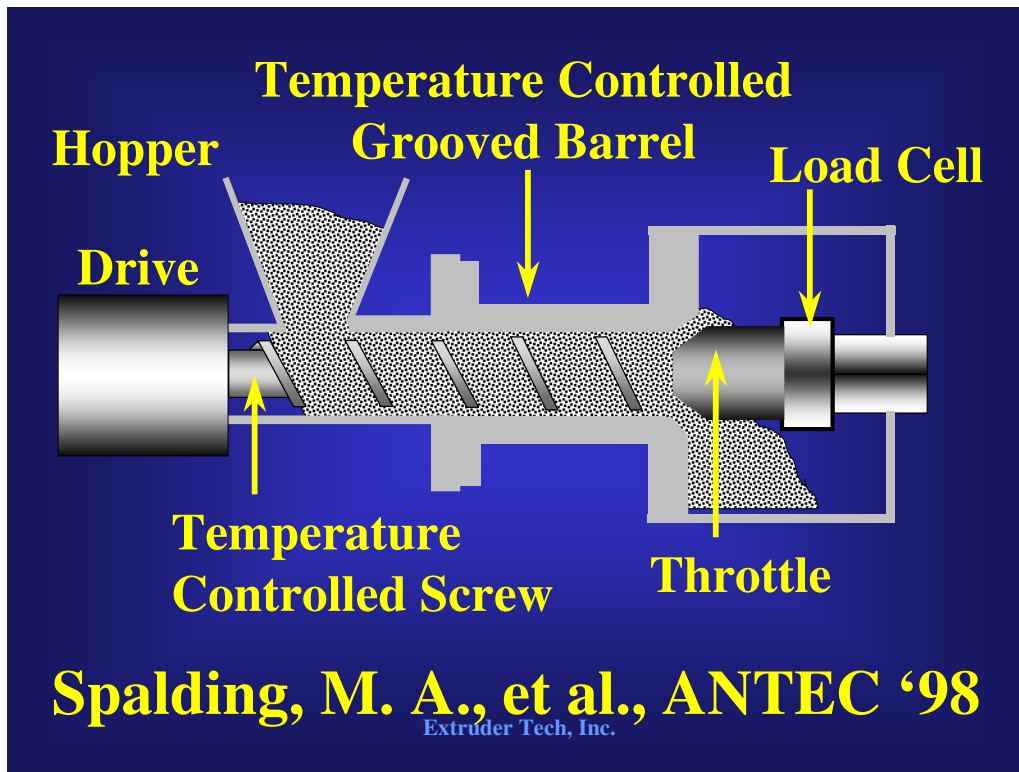
An experimental means is invoked to determine the solids conveying angle for a grooved barrel extruder.

**DATA
Grooved Barrel
LDPE
Flow vs. Pressure**

Spalding, M. A., et al., ANTEC '98

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Accurate data for solids flow versus pressure for a grooved barrel extruder is determined by special experimental means. Such data are available in the literature for LDPE.



The lab device to measure the flow versus pressure for solids conveying is schematically shown. Features are:

- Temperature controlled grooved barrel
- Temperature controlled screw
- Speed controlled drive
- Throttle to restrict flow
- Load cell to measure force and pressure at the exit

Test Screw Dimensions

63.5 mm Diameter

Square Pitch

4.5 L/D

Spalding, M. A., et al., ANTEC '98

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Basic parameters for the test extruder device are given.

Barrel Groove Dimensions

- 8 grooves, equally spaced
- 2.5 mm width, 2.1 mm depth
- Tapered to zero at the end of the barrel, 3 L/D

Spalding, M. A., et al., ANTEC '98

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The barrel is grooved as given.

Test Conditions

- Speeds of 50 rpm and 80 rpm
- Channel Depths of 8.89 mm and 11.1 mm
- 6 Pairs of Barrel and Screw Temperatures

Spalding, M. A., et al., ANTEC '98
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The process parameters tested are typical for solids conveying of LDPE.

Dimensionless Flow Rate at Pressure, p

$$N_Q(p) = \frac{\dot{m}(p)}{(\rho_B(p)\omega\bar{R}Ht_c)}$$

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The dimensionless flow rate as a function of pressure is obtained from the flow at each value of exit pressure.

Bulk density is also included as a function of pressure. This is readily available in the literature.

Solids Conveying Angle at Pressure, p

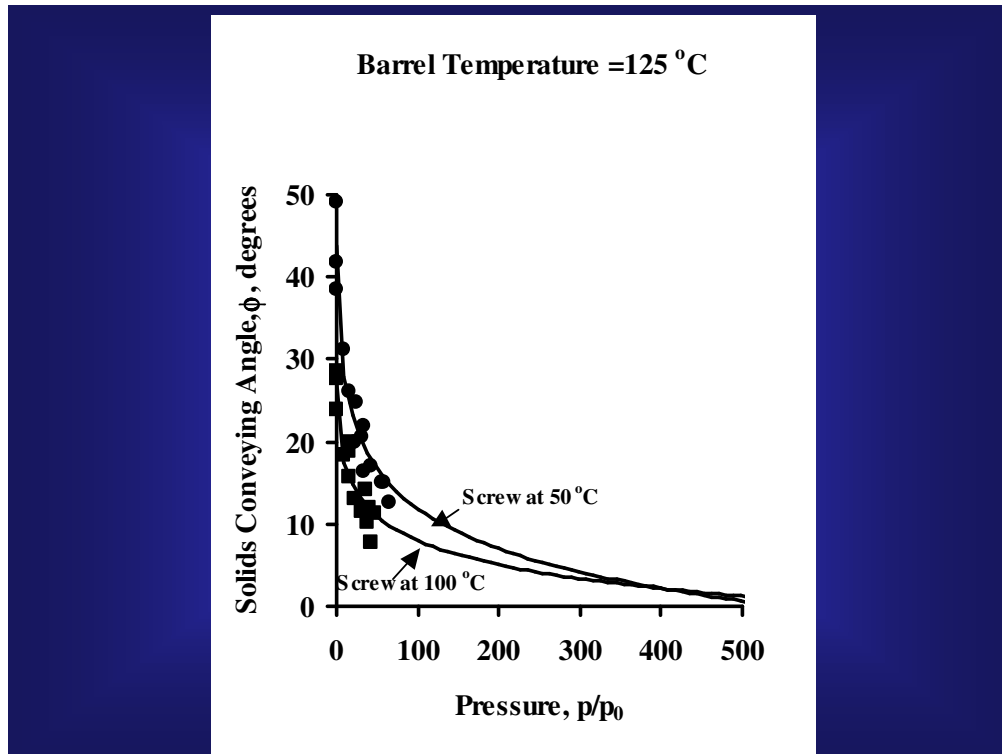
$$\phi(p) = \text{atan} \left[\frac{N_Q(p)\tan(\theta)}{(1-N_Q(p))} \right]$$

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The kinematics flow equation is inverted to solve for the solids conveying angle. Measured flow rates are then used to develop the solids conveying functions of pressure.

This same calculation can be done for any operating extruder if the flow rate can be determined. This can be very valuable in determining if the solids conveying is performing normally, particularly if several machines are being used. The solids conveying angle, as developed for this work, has been shown to be relatively unaffected by screw speed, diameter, or channel depth. Therefore, it makes a useful scaling factor for solids conveying.

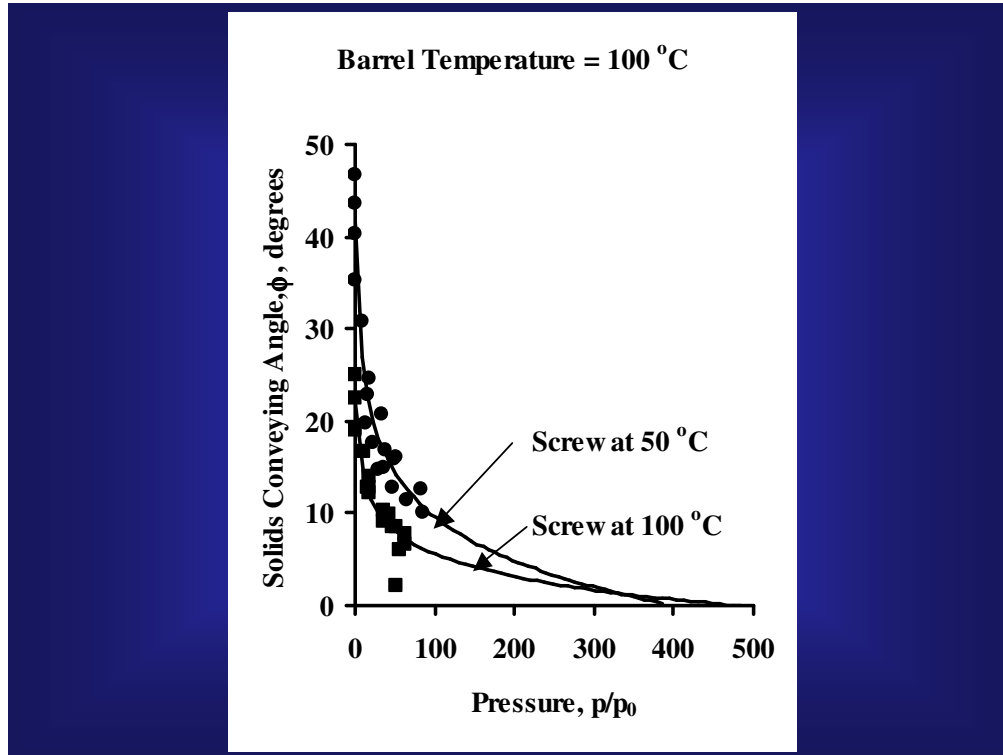
Even though the pressure is not known for an operation extruder, it must be of some consistent value of the process is running well. That is, it must be of proper value to initiate melting at the correct rate. Therefore, the solids conveying pressure will be about the same for every like machine successfully processing the LDPE, so the solids conveying angle should be the same.



Here is the solids conveying results versus pressure ratio. The pressure ratio is the ratio of exit to inlet pressures.

The solids conveying angle here is shown only to be a function of barrel and screw temperatures. Speed and channel depth are not significant factors.

The curves are regression with a logarithmic function.



Here are the results of solids conveying angle for a colder barrel. The logarithmic function is again show to approximate the data.

The solids angle function depends on temperatures.

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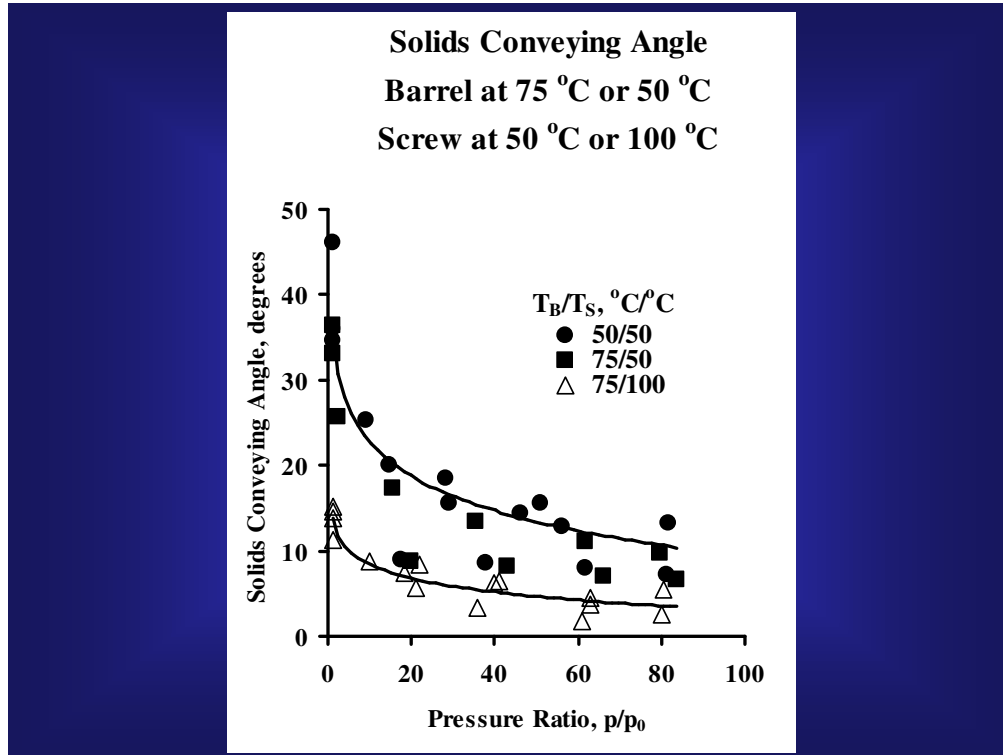
The solids conveying angle has been shown to be a significant function of barrel and screw temperature.

This is akin to saying that the solids conveying angle is a function of barrel and screw friction factors since friction factors are primarily dependent of surface temperature. Therefore, the model includes the variability of friction without the need for knowing the actual values of the the friction factors.

This is a very useful result, as friction factors are very difficult to determine as they exist in an extruder. In the case of a grooved barrel, it is especially difficult to even define the barrel friction factor before even assigning a value to it. Therefore, the kinematics model and solids conveying angle provide an alternative means for analyzing and calculating solids conveying flow in an extruder with a grooved barrel.

**The solids
angle function
is independent
of speed and
channel depth.**

Channel depth and screw speed were found to not be significant factors in the solids conveying angle function.



Here are solids conveying result for temperature conditions that are known to be sub-optimum. The results are bimodal in nature. This indicates that the The bimodal solids conveying angle function indicates that the extruder would tend to have unstable flow since the solids angle could “jump” between two values at any given pressure. The model would then predict a “jump” in flow rate.

The solids angle function is bimodal at non-ideal temperatures.

Non-ideal temperatures produce low flow and unstable operation. The non-ideal barrel temperature is typically not near the melting point of the polymer. The non-ideal screw temperature is usually higher than the barrel temperature or close to the melting point of the polymer.

The barrel friction for a grooved barrel can be considered as the parallel combination of smooth surface friction with the added traction of the grooves. When added together, they form the total tractive force at the barrel wall. So, when solids flow with a grooved barrel falters, consideration should be taken of how conditions on the smooth portion of the barrel surface and at/in the grooves are being affected.

If the tractive force of the screw becomes large enough, it can overcome the total tractive force on the barrel. This can happen by failure of the smooth surface friction or by failure of the grooves.

Failure of the smooth surface friction would be a result of cold temperatures. The polymer does not melt and become sticky.

Failure of the tractive force at the grooves would be shearing of the polymer itself at the groove. This would result in the internal sliding friction of the polymer to provide the tractive force at the barrel grooves.

Therefore, the multiple combinations barrel tractive forces give rise to the bi-modal performance at non-ideal conditions.

**A logarithmic
function of
solids angle vs.
pressure is
assumed.**

The logarithmic function of solids angle versus pressure ratio has basis in the Tadmor and Klein model for solids flow. In it, the function of solids conveying angle is defined by the logarithm of pressure ratio. This adds to the confidence that the logarithmic function is an accurate approximation to the data.

Function of Solids Angle with Pressure

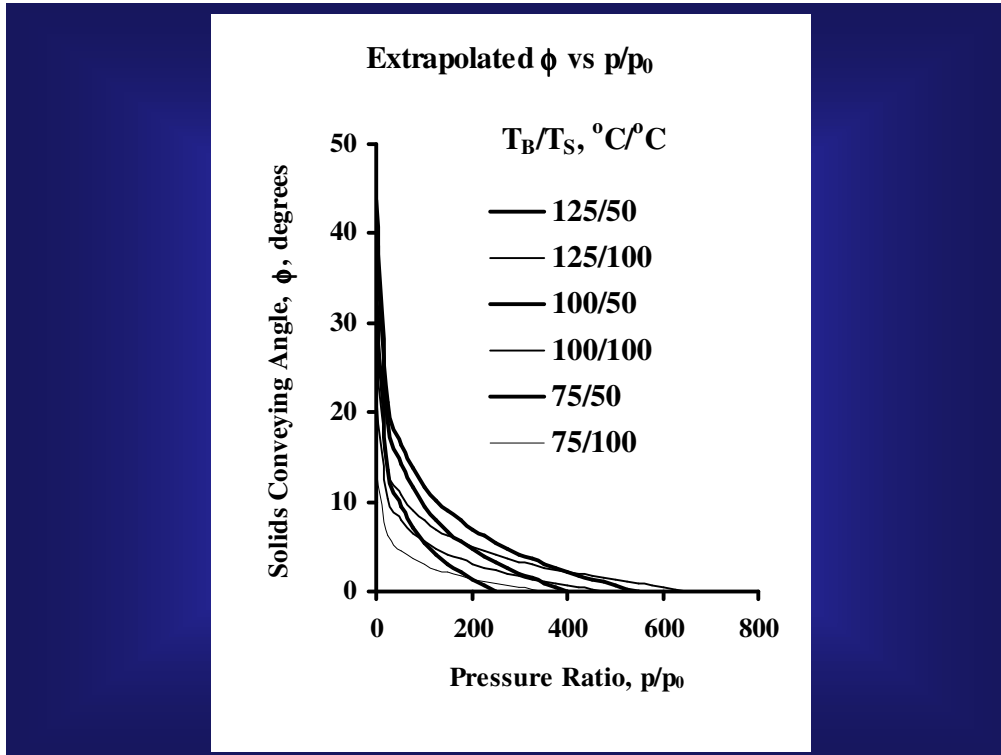
$$\varphi(p) = A \ln(p/p_0) + B$$

**A and B are functions of
Temperatures.**

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The logarithmic function is dependent on barrel and screw temperatures. Factors A and B are a function of barrel and screw temperatures and are determined by regression analysis for each temperature pair.

So, in order to use the results, the barrel and screw temperature must be known. In other words, the barrel and screw friction factors must be set even though they are not evaluated.



Here are all three sets of solids conveying angle results versus pressure ration. It can be seen that the functions are extrapolated to zero pressure, which is far beyond the pressure ratio of the tests. However, it is surprising how well behaved the functions appear.

**The solid angle
log functions
exhibit regular
behavior.**

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This is an unexpected result for such an extrapolation. It provides further confidence that the model is valid and that the logarithmic approximation for the solids conveying angle is accurate.

SOLIDS CONVEYING FLOW VS PRESSURE

Now that the solids angle function vs pressure is known, it can be used with the kinematics equation to numerically predict the solids flow as a function of pressure.

Kinematics Solids Flow vs. Pressure for Grooved Barrel

$$N_Q = \frac{\tan (A \ln(p/p_0)+B)}{(\tan \theta + \tan (A \ln(p/p_0)+B))}$$

**A and B are functions of
Temperatures.**

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Here is the kinematics model with the logarithmic function for solids angle vs. pressure.

As can be seen, barrel and screw temperatures must be known to provide constants A and B.

Flow Rate vs. Pressure

$$\dot{m} = \rho_B(p) \omega \bar{R} H t_c \frac{\tan(A \ln(p/p_0) + B)}{\tan\theta + \tan(A \ln(p/p_0) + B)}$$

**A and B are functions of
Temperatures.**

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The model is reduced to completed form in units of flow.

It is shown that the bulk density is also involved as a function of pressure to complete the model. Bulk density is evaluated at room temperature, and the temperature is assumed not to change in the solids conveying portion of the process.

How well does the kinematics model approximate data?

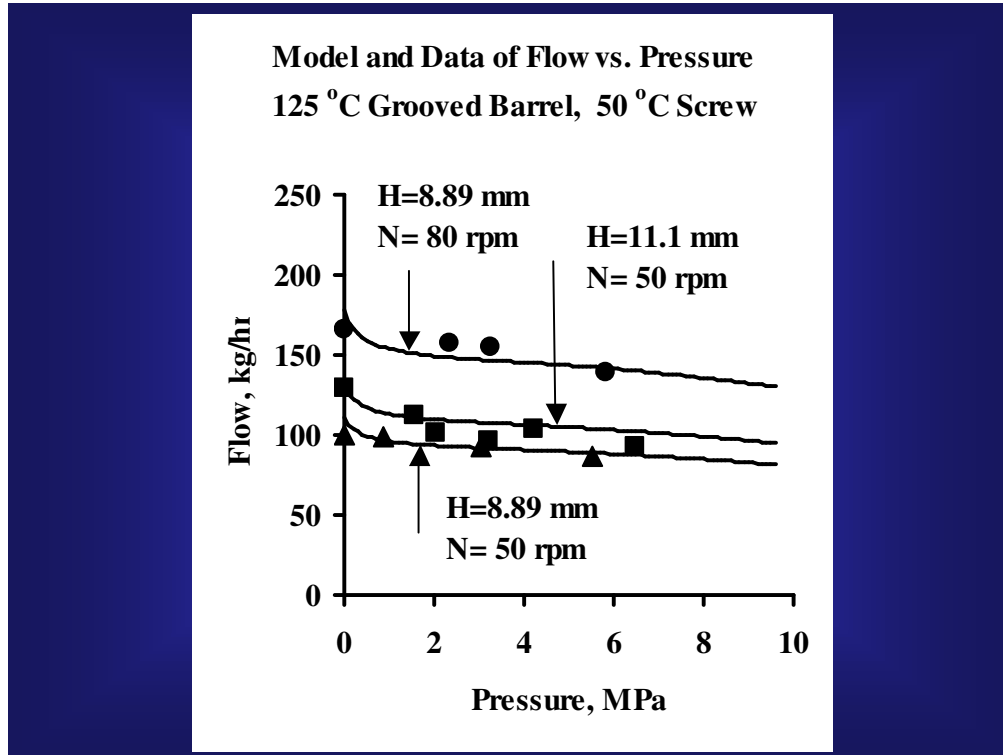
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The model will now be used to recreate the data for comparison.

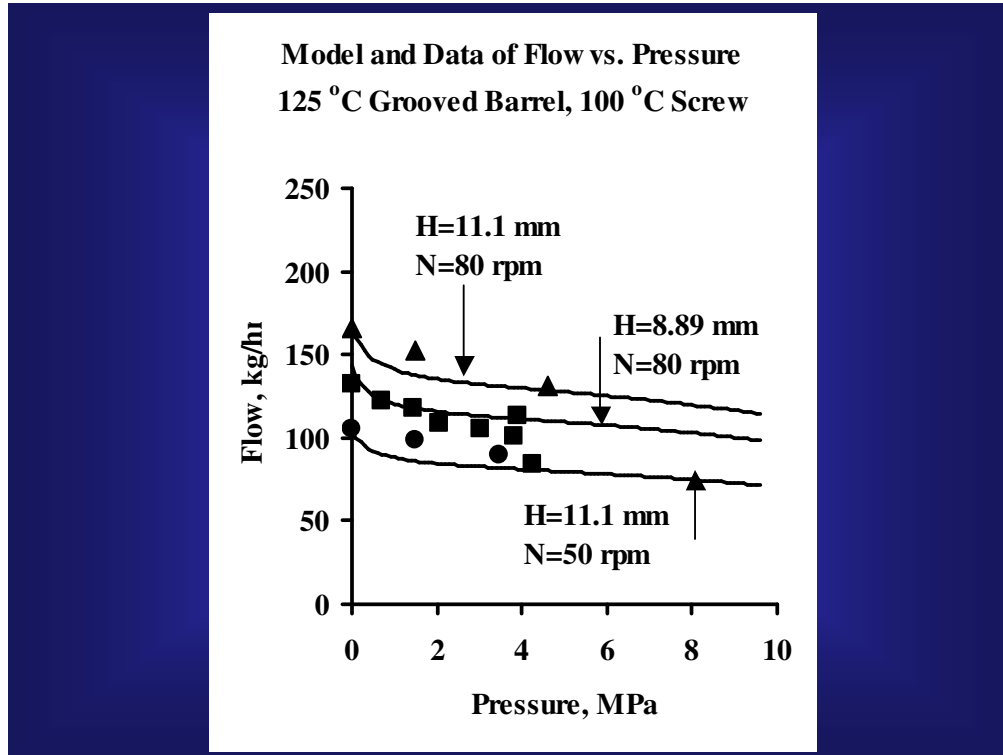
Compare data to model for flow versus pressure.

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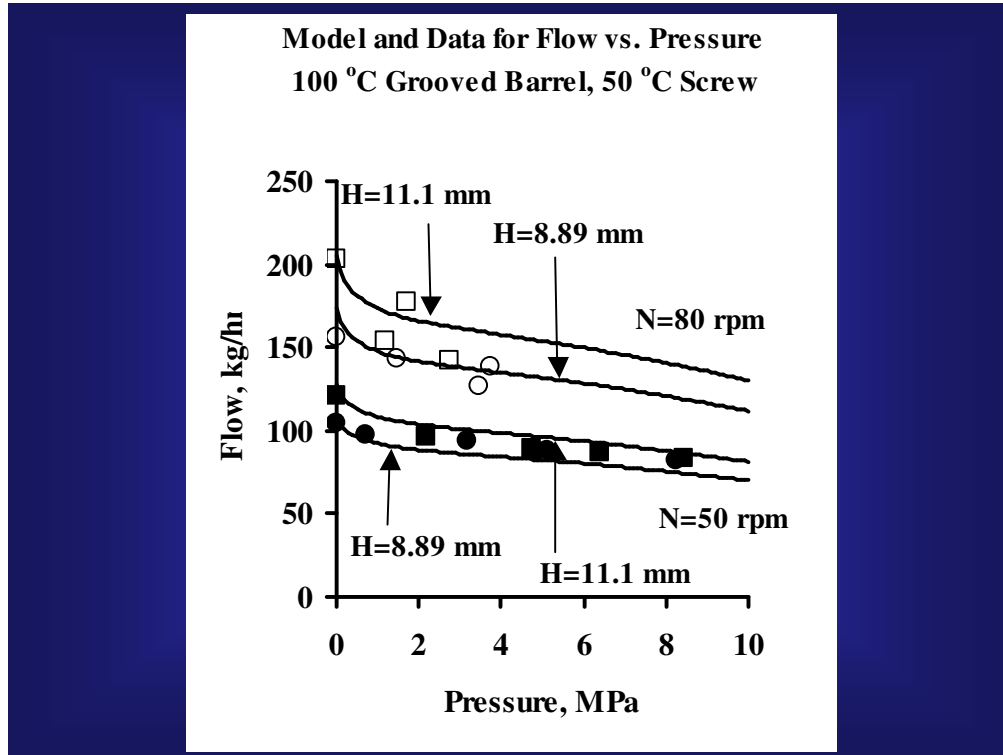
The data of flow versus pressure is now plotted with the predictions of the model defined with the logarithmic function of solids conveying angle versus pressure ratio.



Here are results for optimum operating temperatures. The lines are predicted by the model, and data for two screws and two speeds are shown.

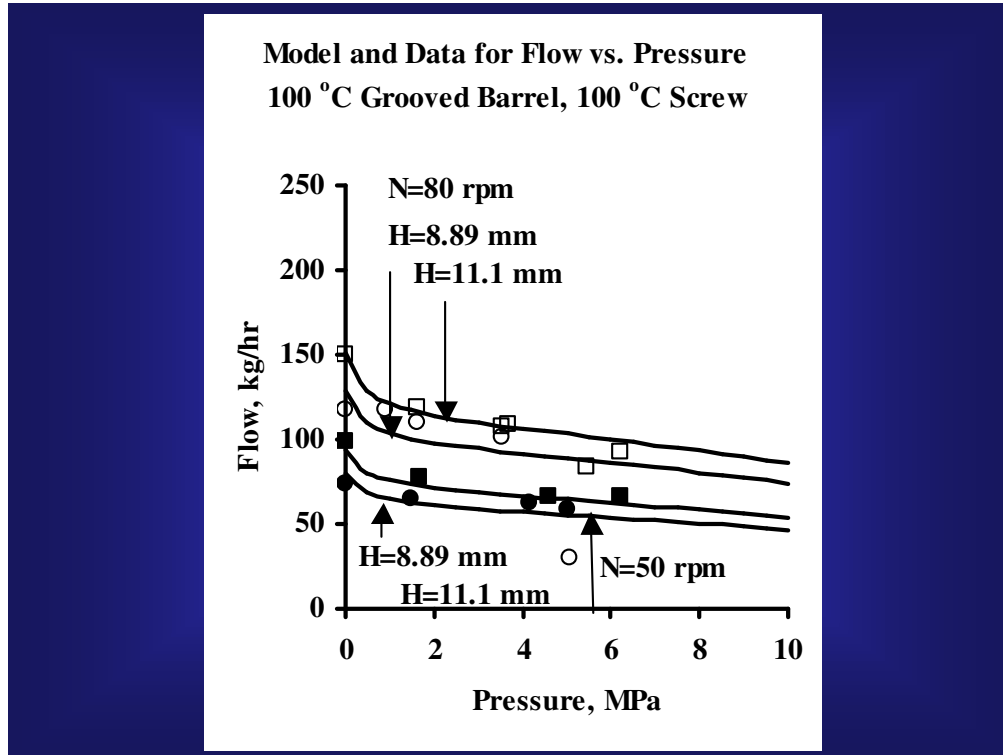


Here are data and model for a different set of screw and barrel temperatures. Again, two channel depths and two speeds are shown. Lines are the predicted flow from the kinematics model.



Here, the barrel temperature is lowered. The data and model are in good agreement.

In fact, the model predicts that the flow for the deeper channel is greater than the flow for shallower channel at 50 rpm. The data do not show this difference. Logically, the flow should be greater for a deeper channel. The model makes this discrimination that the data do not.



Again, the model discriminates the flow between the two screw depths with resolution that the data do not exhibit. The temperature conditions for this case are not optimum in that barrel and screw are the same. Normally, the barrel should be warmer than the screw to promote a greater friction there. Greater friction on the barrel is typically key to good solids conveying.

CONCLUSIONS

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**The kinematics model is
an accurate and robust
means for calculating
solids conveying flow in a
grooved barrel extruder.**

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A grooved barrel extruder is very difficult to analyze with classical barrel/screw friction factor models. Using the solids conveying angle as the constitutive variable avoids the complex issue of defining the “friction factor” for a grooved barrel.

**The kinematics model
does not directly use
friction factors, which are
difficult to define for a
grooved barrel.**

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However, the effect of friction and changes in friction is represented through the barrel and screw temperatures.

**The solids conveying
angle, as a single
constitutive variable,
takes the place of friction
in the model.**

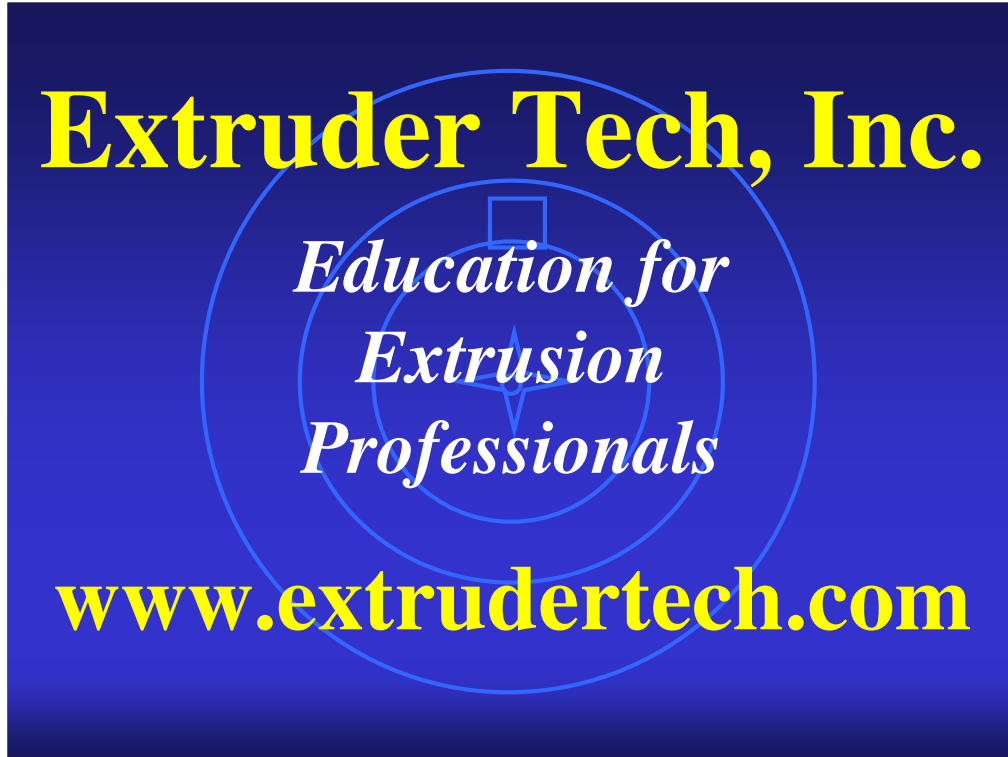
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The solids conveying angle is a function of barrel and screw temperatures. Therefore, it changes as a function of these two independent variables which also have a direct effect on the barrel and screw friction.

**Specification of barrel
and screw temperatures
for the solids conveying
angle function account
for variability in friction
factors.**

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The solids conveying angle emerges as a single constitutive variable in the kinematics flow model of solids conveying. The result is a very robust model of solids conveying that closely mimics the data.



Slides and spread sheet calculations are available for download at the web site.