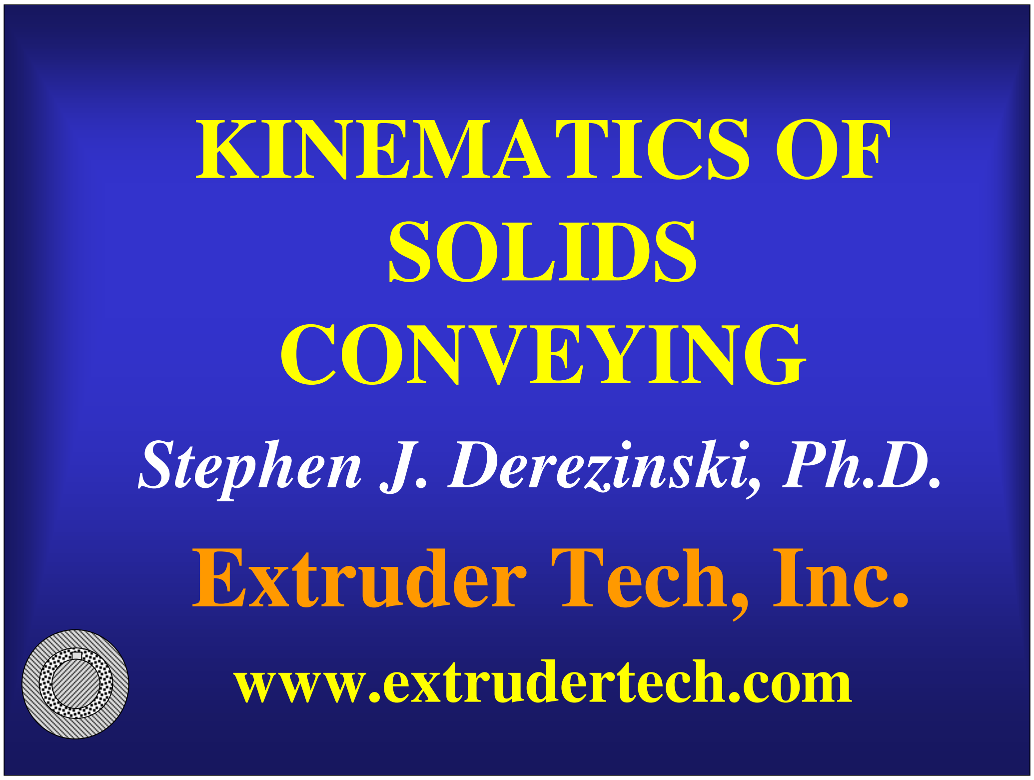

KINEMATICS OF SOLIDS CONVEYING

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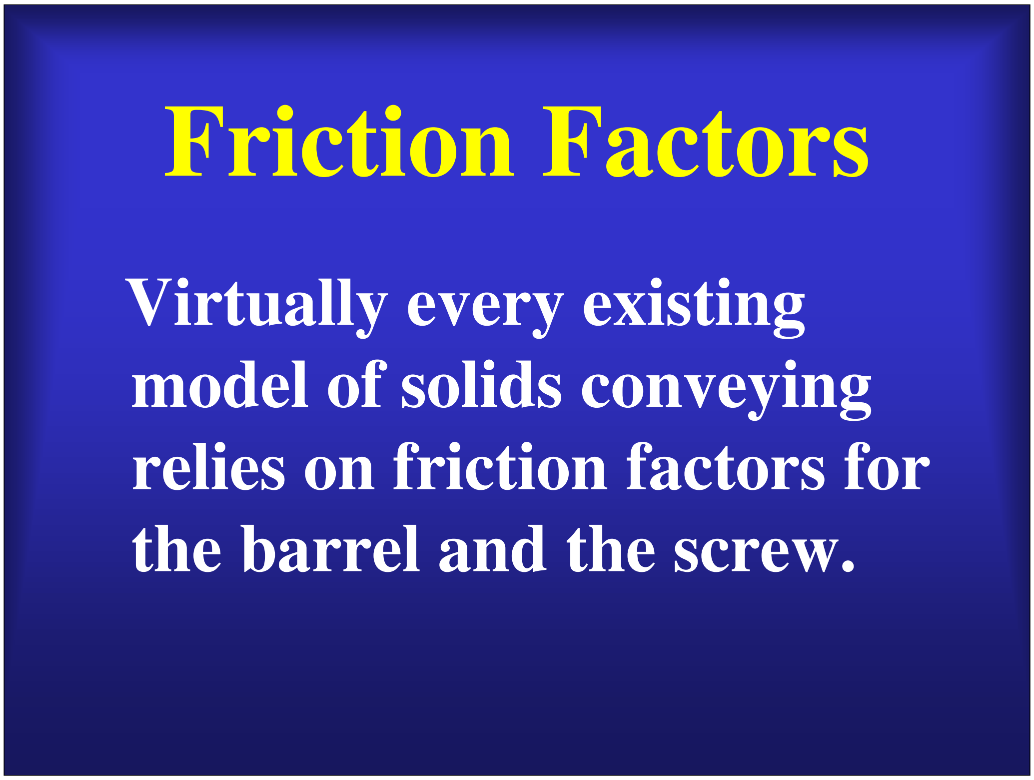
Extrusion is an intriguing combination of a balance of science, math, and art. It seems that practical advances in the analysis and design of the process typically contain such a combination.

It is in the spirit of this observation that the kinematics of the solids conveying has been developed. Relationships of process parameters to flow have been discovered and developed that serve to describe the physics of the process very well. The kinematics provide the mathematical foundation on which to correlate the required constitutive relationships from extruder operation.



Friction Factors

Virtually every existing model of solids conveying relies on friction factors for the barrel and the screw.

To begin, a look at the nature of friction factors is appropriate.

Friction factors are normally used for modeling and understanding the flow in the solids conveying section. Many models have been developed based on a barrel friction factor and a separate screw friction factor.

Friction Factors

- Difficult to measure
- Double valued (barrel and screw factors needed)
- Poorly reproduced

However, these friction factors are difficult to determine. Typically, they are measured with a friction testing machine. That is, sliding friction under a normal load on a flat plate.

The actual process conditions in an extruder are actually quite removed from those of the sliding flat plate. The solids conveying section of an extruder is a helical channel with walls, is curved (cylindrical), and the pressure against the rubbing surfaces (barrel and screw) is orthogonal to the pressure in the flow direction. These are just a few factors that complicate the application of sliding friction phenomena to solids conveying flow.

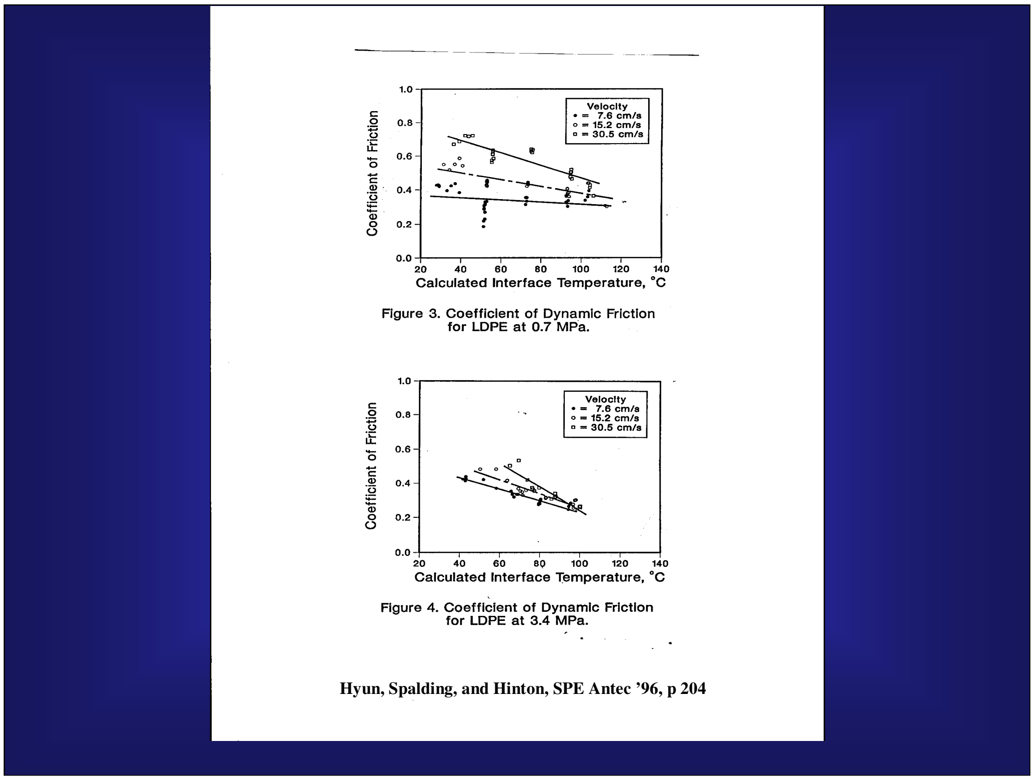
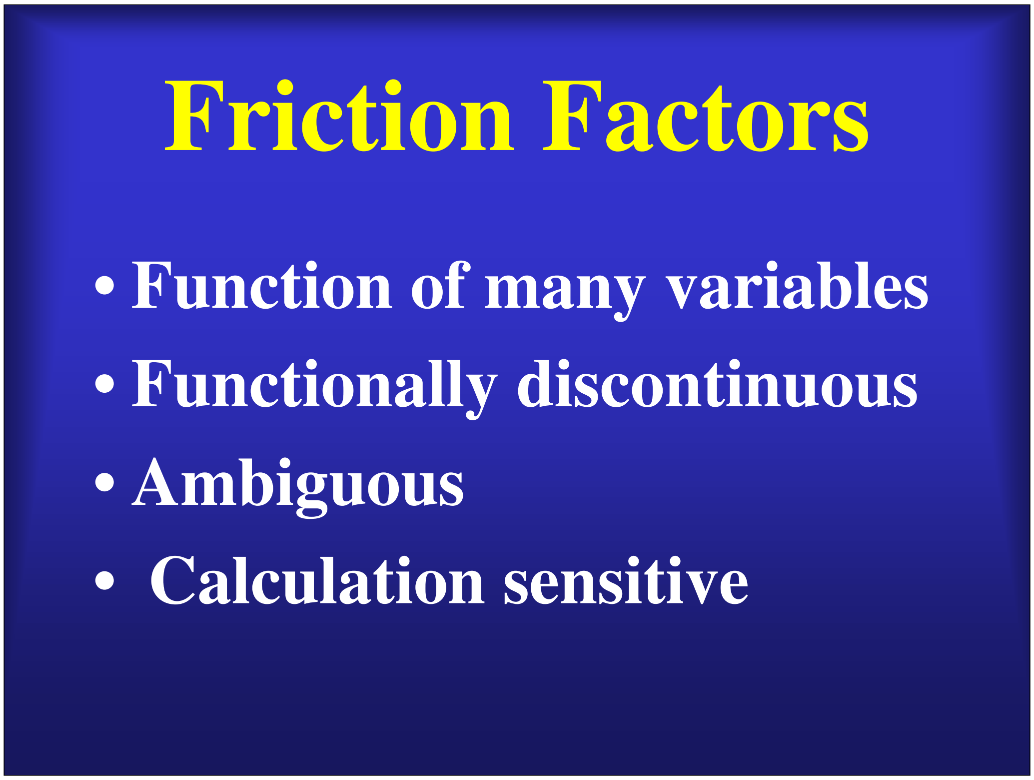


Figure 3. Coefficient of Dynamic Friction for LDPE at 0.7 MPa.

Figure 4. Coefficient of Dynamic Friction for LDPE at 3.4 MPa.

Hyun, Spalding, and Hinton, SPE Antec '96, p 204

Here are some friction factor measurements made for LDPE resin. Scatter of the data is rampant. The data at some conditions appear almost random. Obviously, it is difficult to use such data for accurate calculations.

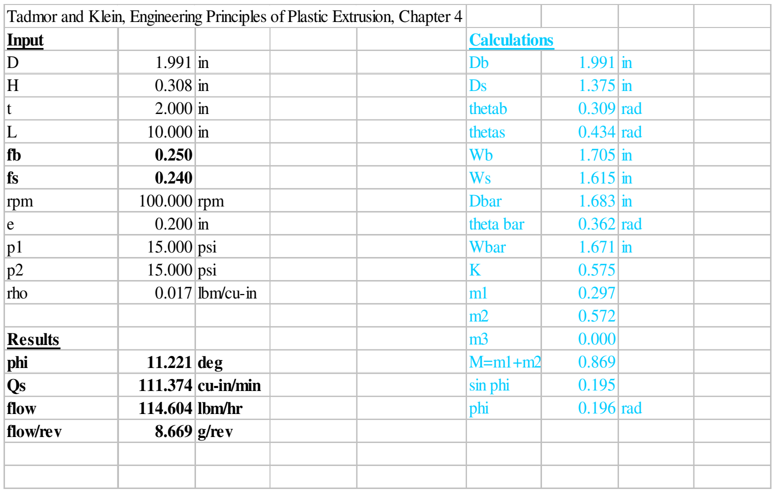


Friction Factors

- Function of many variables
- Functionally discontinuous
- Ambiguous
- Calculation sensitive

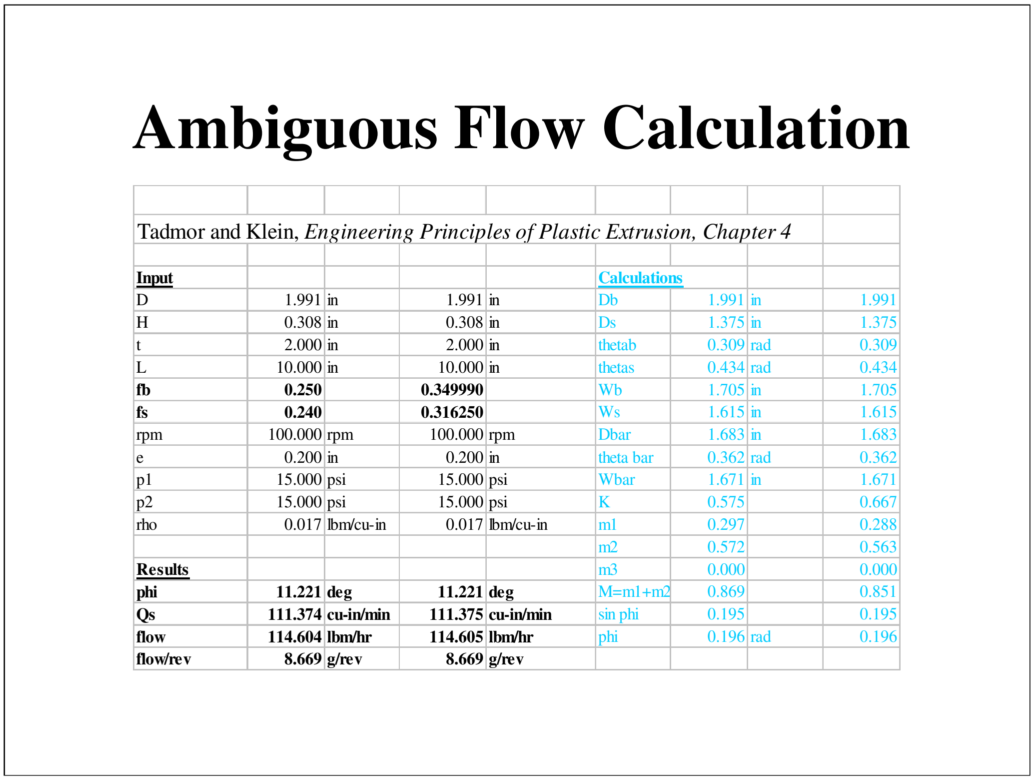
Friction factors are known to be a very complicated variable. The application of such data taken from a sliding plate and applying it to a helical passage compounds the complexity greatly.

Friction Factor Flow Calculation



Tadmor and Klein, Engineering Principles of Plastic Extrusion, Chapter 4					
Input			**Calculations**		
D	1.991	in	Db	1.991	in
H	0.308	in	Ds	1.375	in
t	2.000	in	thetab	0.309	rad
L	10.000	in	thetas	0.434	rad
fb	**0.250**		Wb	1.705	in
fs	**0.240**		Ws	1.615	in
rpm	100.000	rpm	Dbar	1.683	in
e	0.200	in	theta bar	0.362	rad
p1	15.000	psi	Wbar	1.671	in
p2	15.000	psi	K	0.575	
rho	0.017	lbm/cu-in	m1	0.297	
			m2	0.572	
Results			m3	0.000	
phi	**11.221**	**deg**	M=m1+m2	0.869	
Qs	**111.374**	**cu-in/min**	sin phi	0.195	
flow	**114.604**	**lbm/hr**	phi	0.196	rad
flow/rev	**8.669**	**g/rev**			

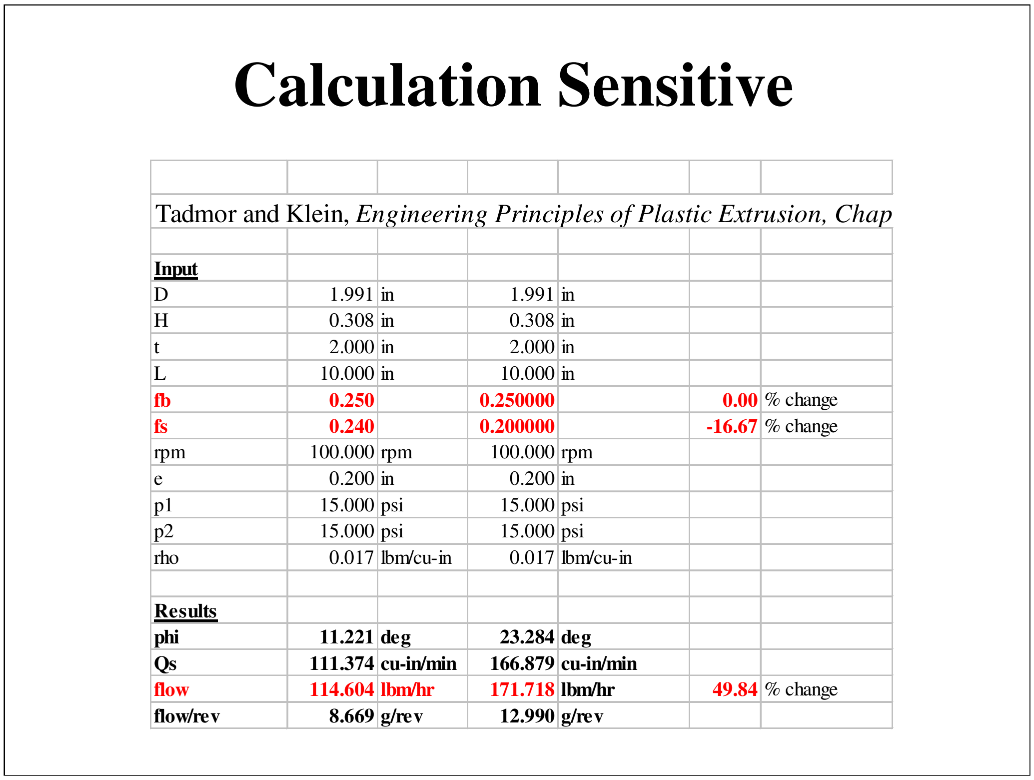
Here is a spread sheet of the Tadmor and Klein model for solids conveying. The values on the right are the intermediate results for completing the calculation of the solids flow.



Ambiguous Flow Calculation

Tadmor and Klein, *Engineering Principles of Plastic Extrusion, Chapter 4*								
Input					**Calculations**			
D	1.991	in	1.991	in	Db	1.991	in	1.991
H	0.308	in	0.308	in	Ds	1.375	in	1.375
t	2.000	in	2.000	in	thetab	0.309	rad	0.309
L	10.000	in	10.000	in	thetas	0.434	rad	0.434
fb	**0.250**		**0.349990**		Wb	1.705	in	1.705
fs	**0.240**		**0.316250**		Ws	1.615	in	1.615
rpm	100.000	rpm	100.000	rpm	Dbar	1.683	in	1.683
e	0.200	in	0.200	in	theta bar	0.362	rad	0.362
p1	15.000	psi	15.000	psi	Wbar	1.671	in	1.671
p2	15.000	psi	15.000	psi	K	0.575		0.667
rho	0.017	lbm/cu-in	0.017	lbm/cu-in	m1	0.297		0.288
					m2	0.572		0.563
Results					m3	0.000		0.000
phi	**11.221**	**deg**	**11.221**	**deg**	M=m1+m2	0.869		0.851
Qs	**111.374**	**cu-in/min**	**111.375**	**cu-in/min**	sin phi	0.195		0.195
flow	**114.604**	**lbm/hr**	**114.605**	**lbm/hr**	phi	0.196	rad	0.196
flow/rev	**8.669**	**g/rev**	**8.669**	**g/rev**				

Here is an interesting aspect of the solids flow equations. Two calculations are done for the same process conditions, but different friction factors. By a trial-and-error method, different sets of friction factors were found that give the exact same results. So, it can be concluded that there is an infinite number of combinations of barrel and screw friction factors that will produce the same flow and pressure. This could be quite misleading under the right circumstances.

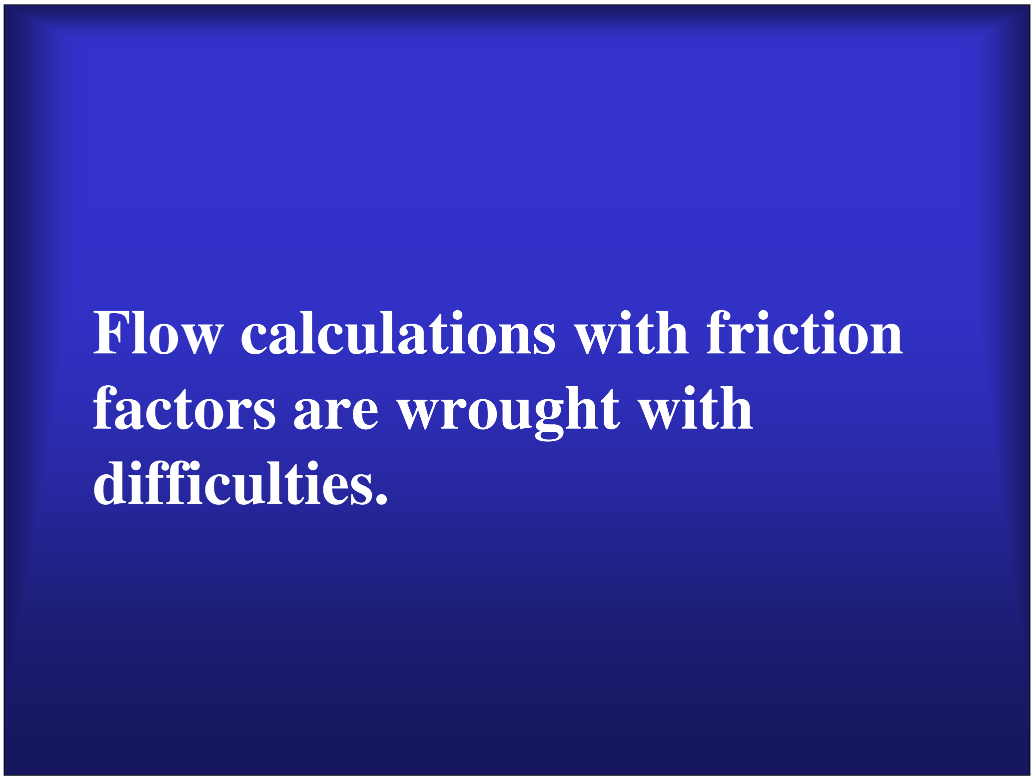


Calculation Sensitive

Tadmor and Klein, *Engineering Principles of Plastic Extrusion, Chap*

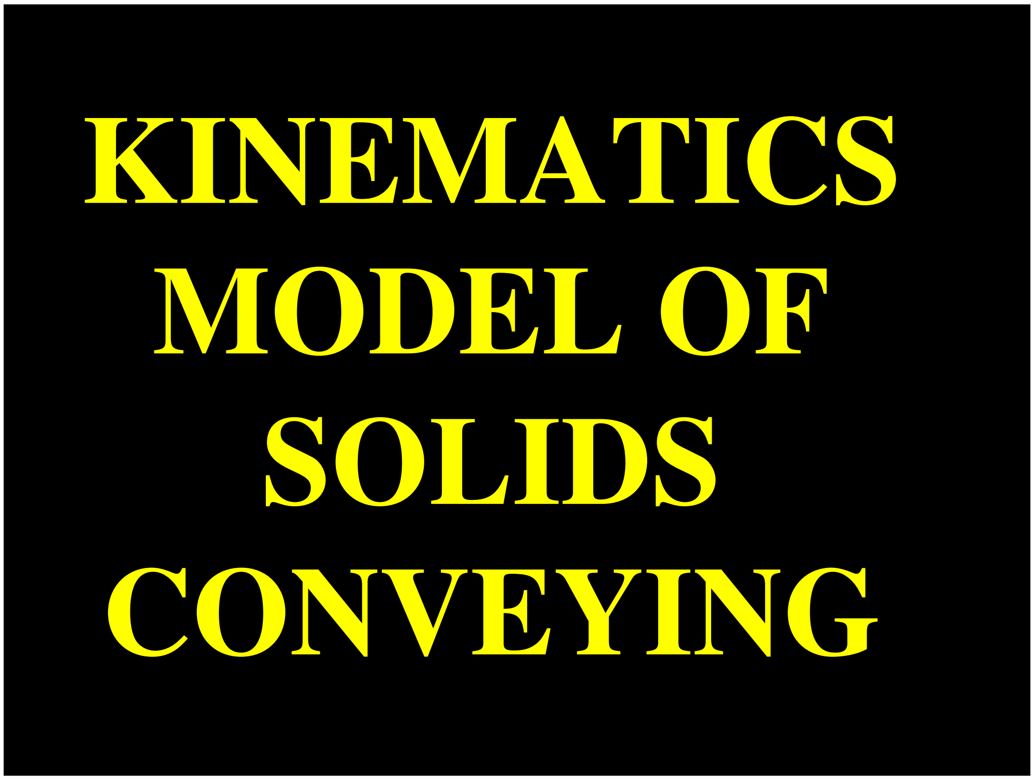
Input						
D	1.991	in	1.991	in		
H	0.308	in	0.308	in		
t	2.000	in	2.000	in		
L	10.000	in	10.000	in		
fb	**0.250**		**0.250000**		**0.00**	% change
fs	**0.240**		**0.200000**		**-16.67**	% change
rpm	100.000	rpm	100.000	rpm		
e	0.200	in	0.200	in		
p1	15.000	psi	15.000	psi		
p2	15.000	psi	15.000	psi		
rho	0.017	lbm/cu-in	0.017	lbm/cu-in		
Results						
phi	**11.221**	**deg**	**23.284**	**deg**		
Qs	**111.374**	**cu-in/min**	**166.879**	**cu-in/min**		
flow	**114.604**	**lbm/hr**	**171.718**	**lbm/hr**	**49.84**	% change
flow/rev	**8.669**	**g/rev**	**12.990**	**g/rev**		

Here is another look a the Tadmor and Klein model that demonstrates how sensitive the results are to a the friction factor values. Notice that for a 16% change in friction factor that a 49% change in flow is calculated. Normally, solids conveying is much more stable than calculated here.



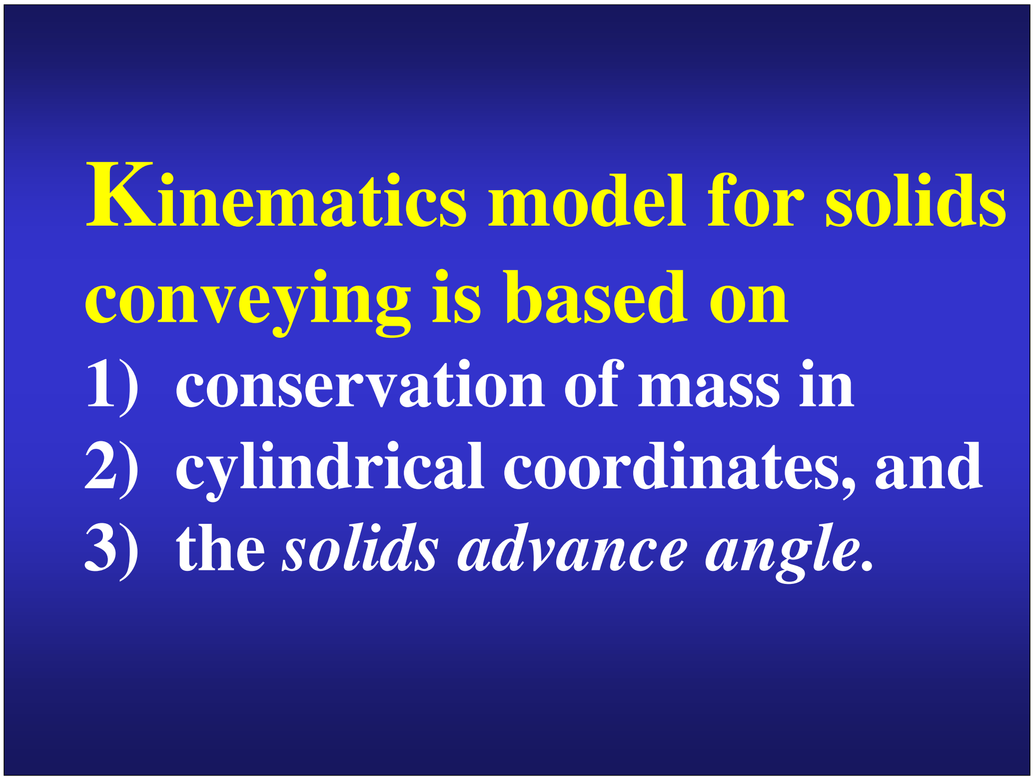
Flow calculations with friction factors are wrought with difficulties.

The conclusion is reached that friction factor models are difficult to administer and can be misleading. This is apparent from other models of this ilk beside the Tadmor and Klein model.



KINEMATICS MODEL OF SOLIDS CONVEYING

Through the analysis of the kinematics of the solids plug flow, relationships have been discovered that can be used to understand and calculate the performance of solids conveying. The knowledge of the friction factors is not needed. However, the method does not disregard that they are involved in the process or contradict conventional theory.

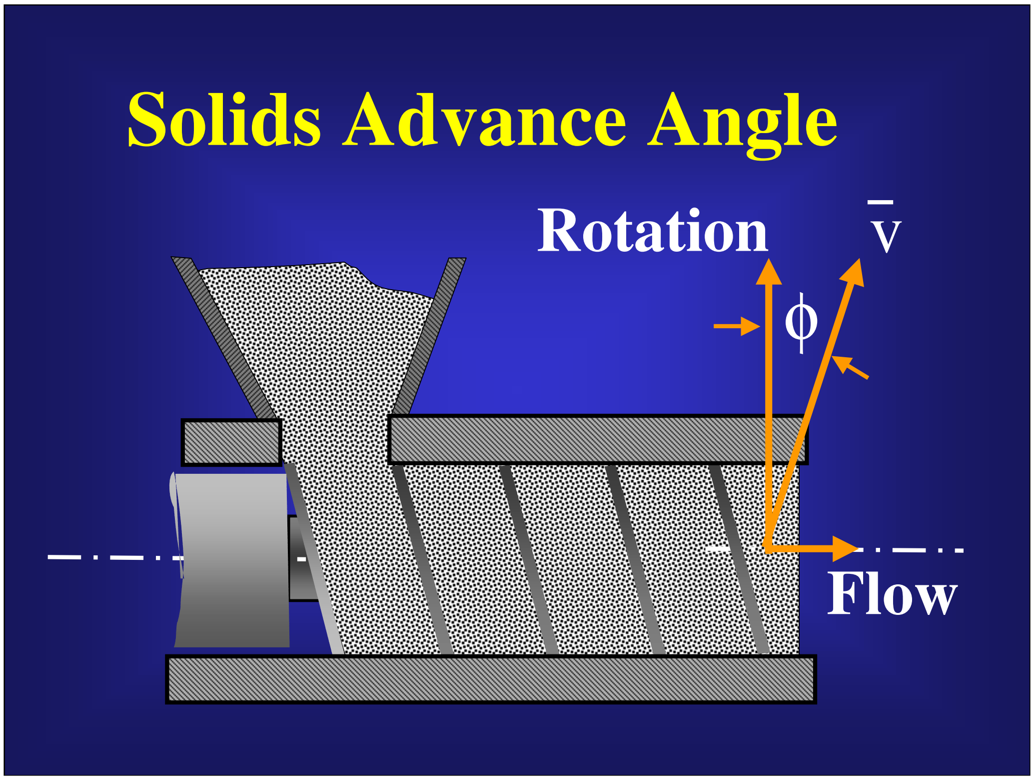


Kinematics model for solids conveying is based on

1) conservation of mass in
2) cylindrical coordinates, and
3) the *solids advance angle*.

The conservation of mass in cylindrical coordinates is used to define the solids conveying angle for the solids plug. The solids conveying angle then becomes a single constitutive parameter that replaces the barrel and screw friction factors. In effect, the solids conveying angle will be shown to replace the two friction factor values.

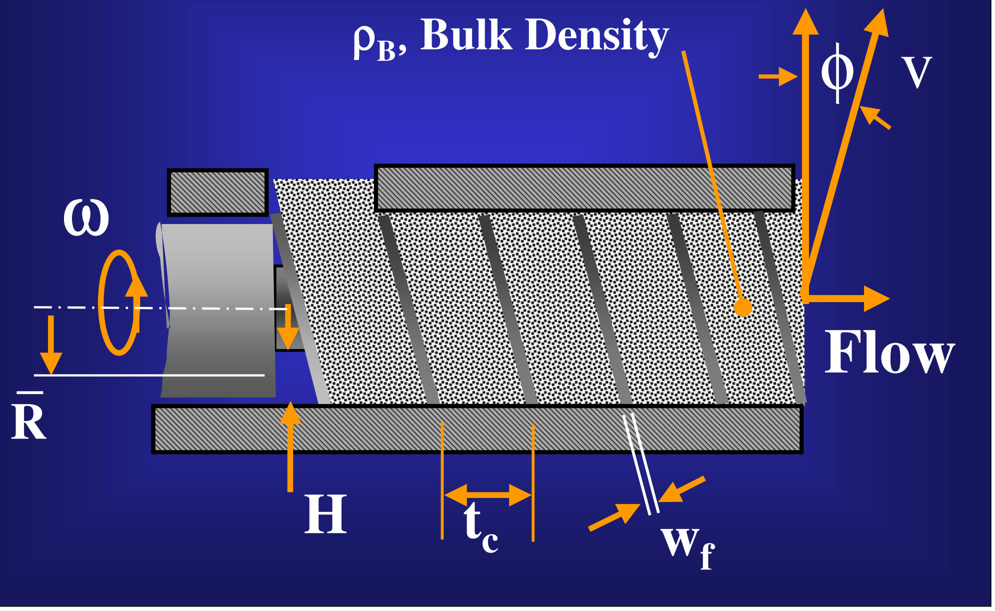
However, it will be shown that the solids conveying angle is a function of the conditions at the *screw and barrel surface*. Therefore, the conditions on the barrel and the screw are needed for the kinematics model are just the same as for any friction factor model. Therefore, the effect of friction factors is fully included in the kinematics model, but the calculation of them is avoided by use of the solids conveying angle as a single constitutive parameter.



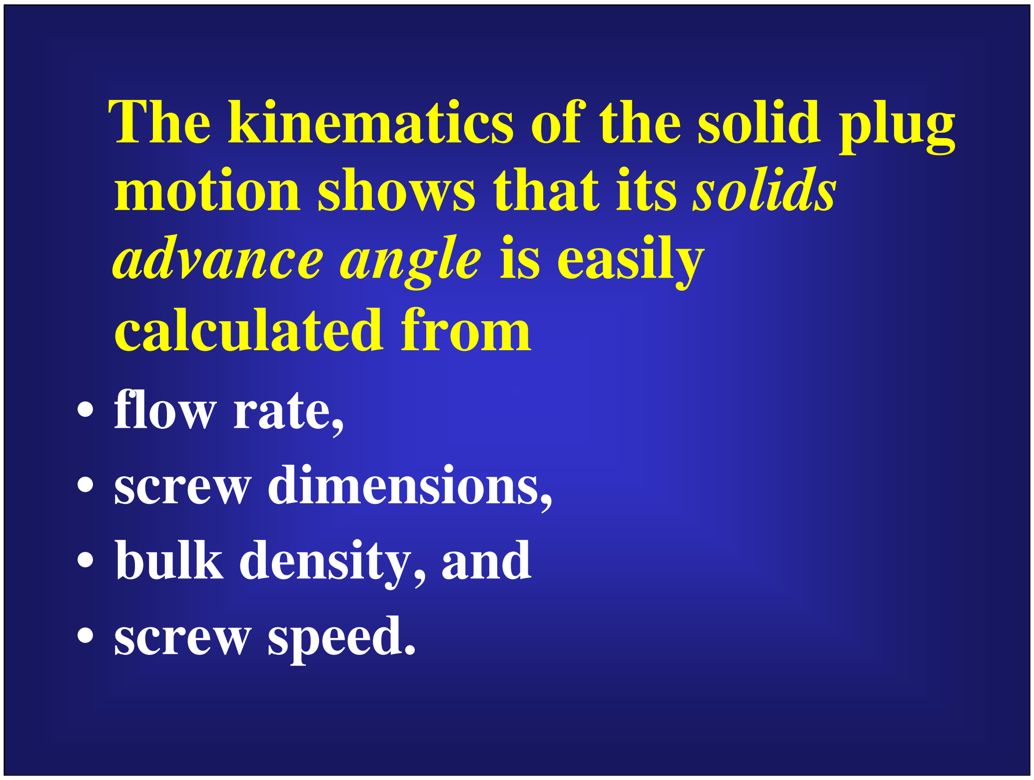
Solids Advance Angle

Here is the definition of the solids advance angle. It is defined at the end of the solids conveying portion of the extruder..

Parameters



Here are the process variables that are needed to define the solids conveying angle. They were determined from the kinematics equation for the solids plug.

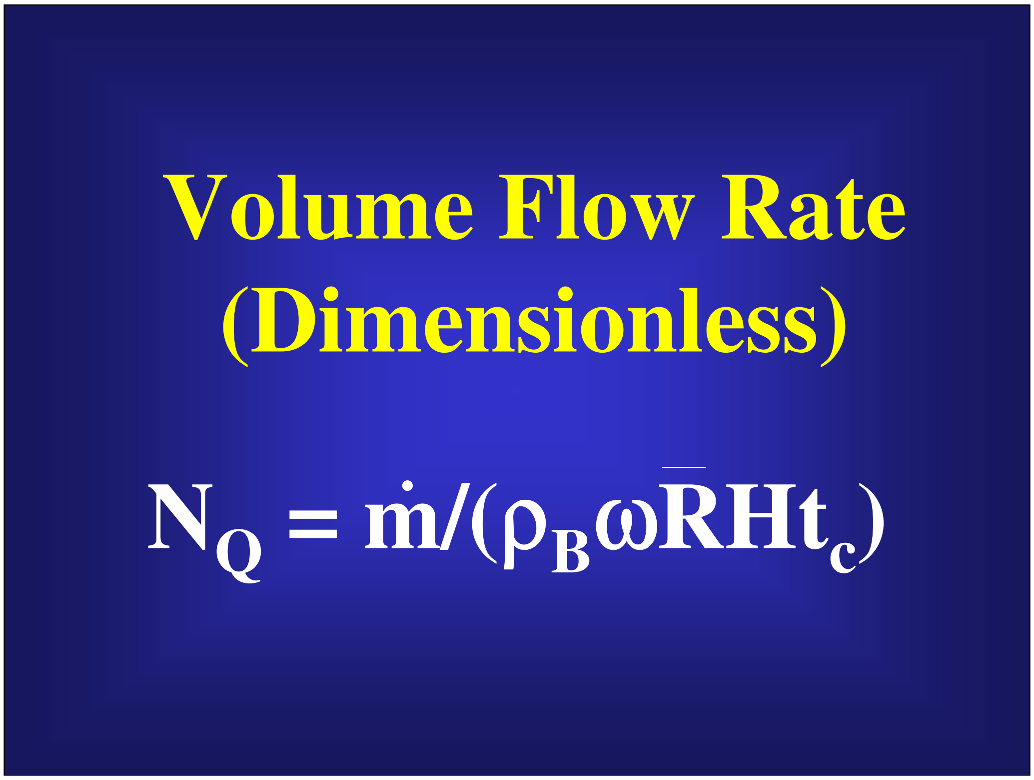


The kinematics of the solid plug motion shows that its *solids advance angle* is easily calculated from

- flow rate,
- screw dimensions,
- bulk density, and
- screw speed.

All of these variable can be easily and accurately determined.

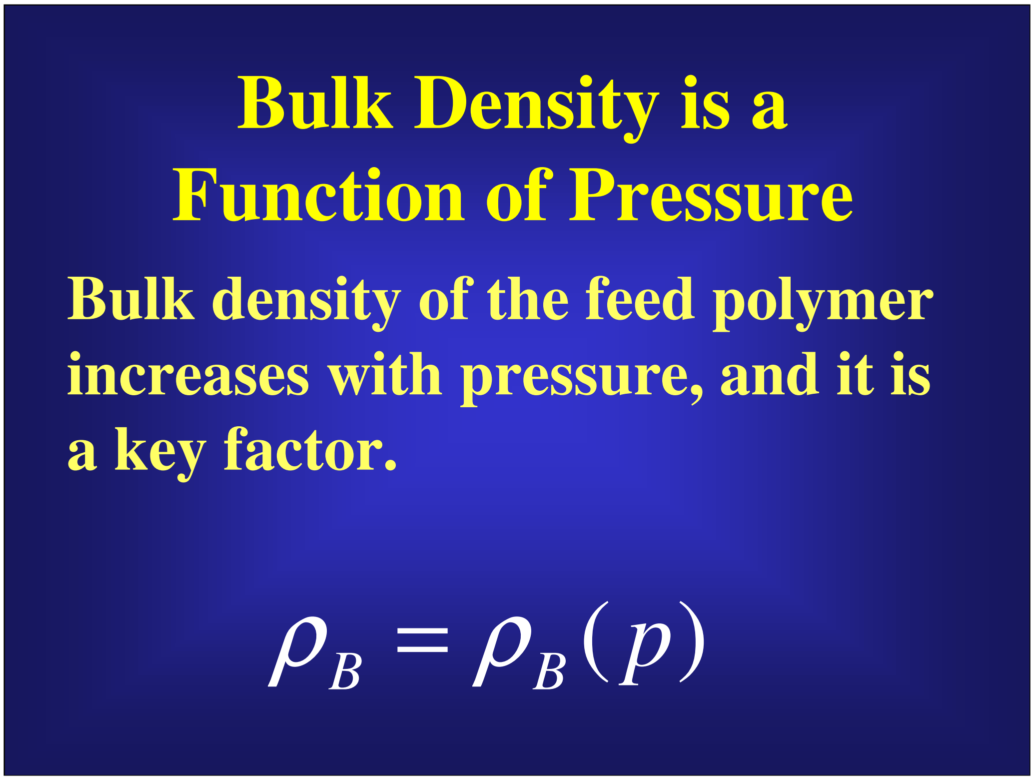
All of the factors are easily obtained from a machine in operation without special instrumentation. So, the solids angle can be obtained for production machines, which is a very useful aspect of the method.



Volume Flow Rate (Dimensionless)

$$N_Q = \dot{m}/(\rho_B \omega \bar{R} H t_c)$$

The dimensionless volume flow rate that the kinematics model defines is used extensively for the method.

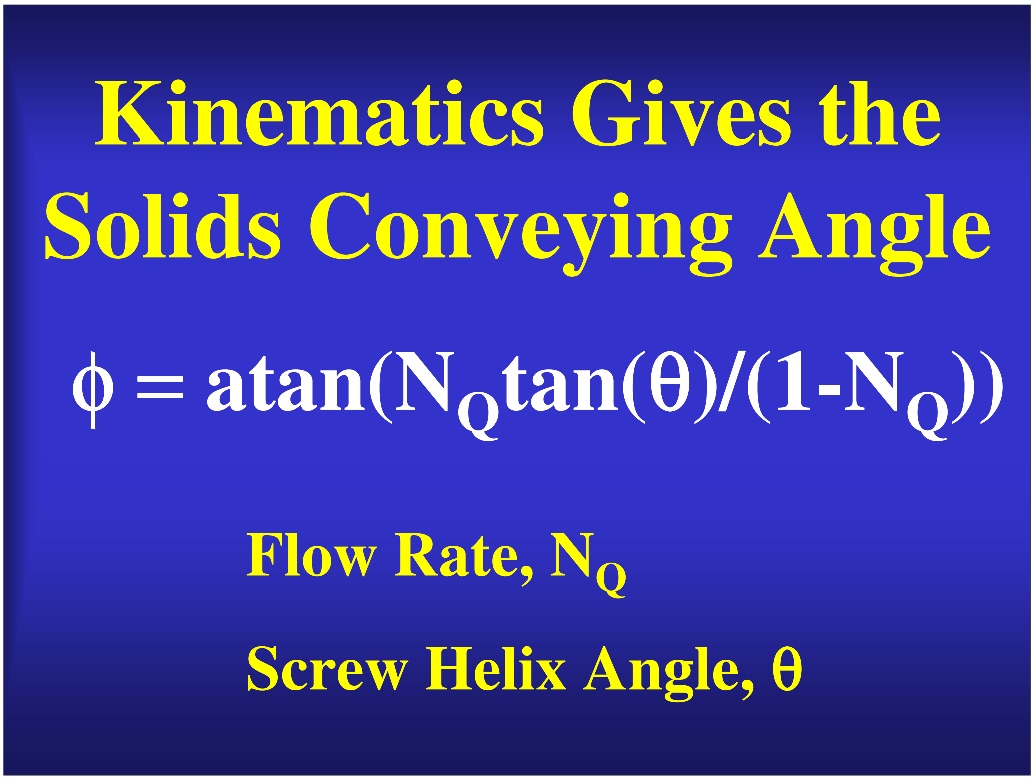


Bulk Density is a Function of Pressure

Bulk density of the feed polymer increases with pressure, and it is a key factor.

$$\rho_B = \rho_B(p)$$

Bulk density of the polymer resin as a function of density is the one important polymer parameter needed by the method. For virgin materials, it can be found in the literature. For mixtures, such as blends or resin with re-cycle, measurements must be made.



Kinematics Gives the Solids Conveying Angle

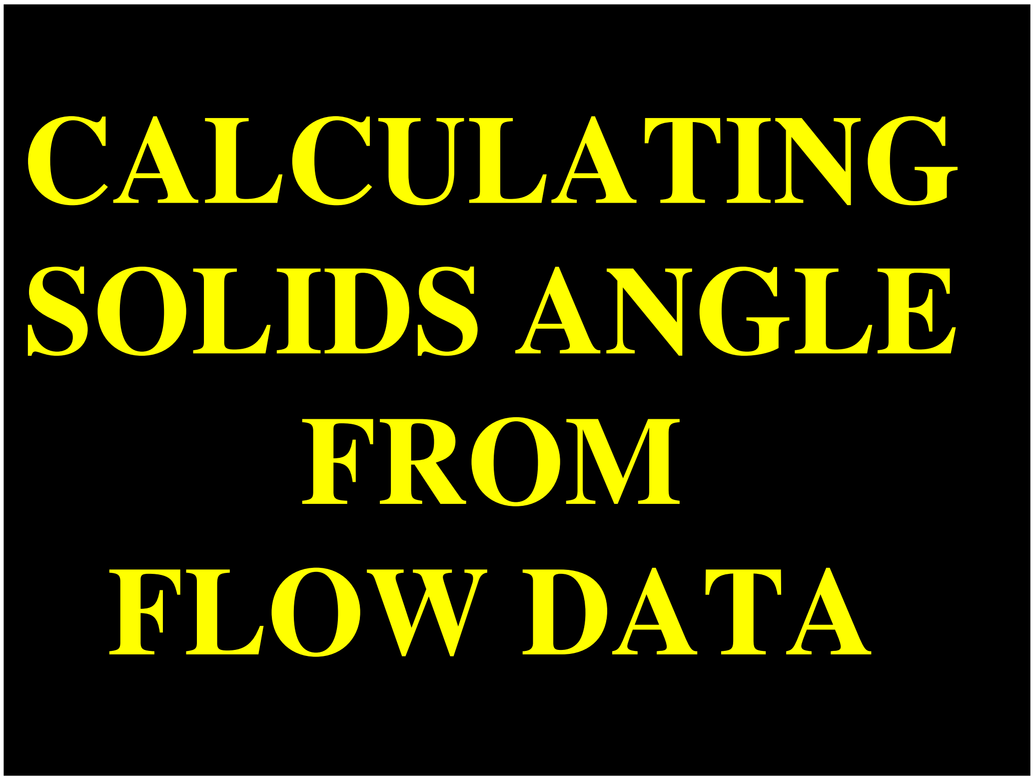
$$\phi = \mathrm{atan}(N_Q \tan(\theta)/(1-N_Q))$$

Flow Rate, N_Q

Screw Helix Angle, θ

The kinematics model provides the above equation for the solids angle based on the previously defined dimensionless flow rate and the helix angle of the screw.

Again, it is important to note that cylindrical coordinates are used to calculate the solids angle and this includes the helix angle.

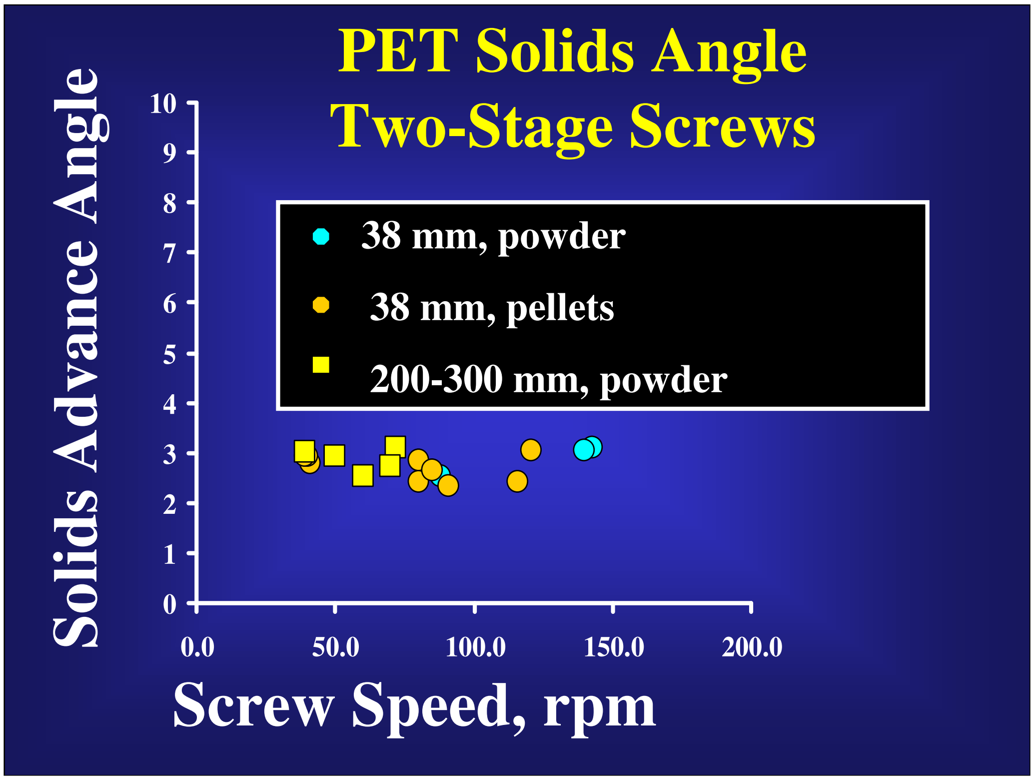


CALCULATING SOLIDS ANGLE FROM FLOW DATA

Flow data from various machines are used to calculate the solids angle.

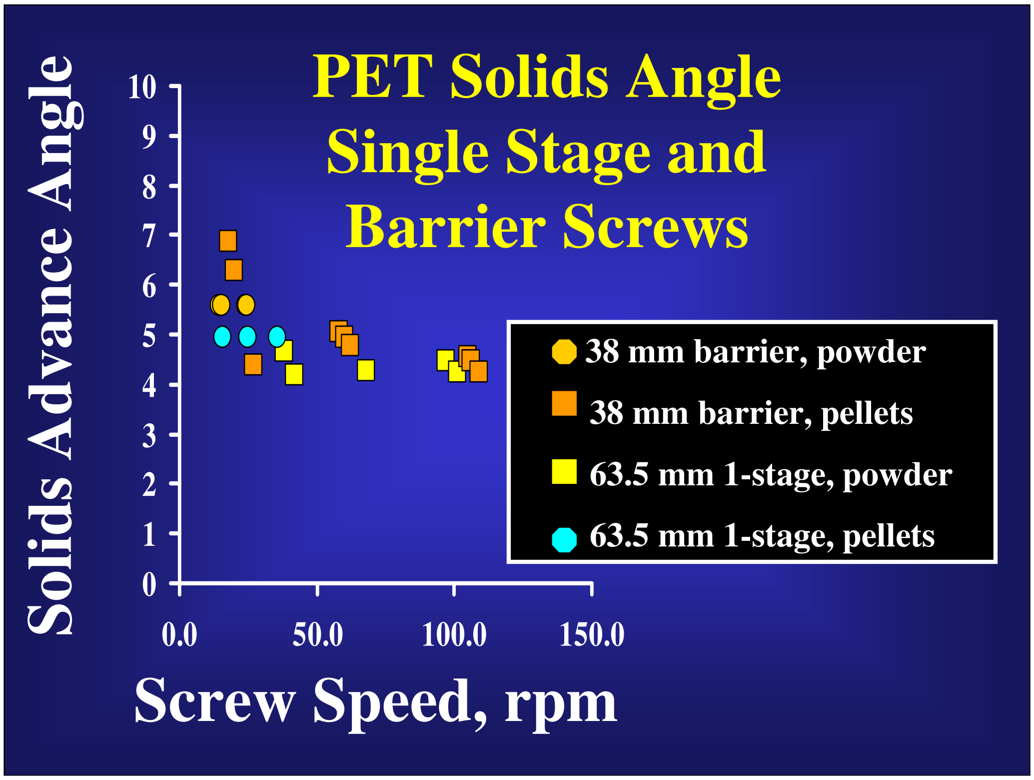
PET DATA

Data for PET are shown first.



PET Solids Angle
Two-Stage Screws

Data for the solids angle for several machines with PET pellets or powder show that the solids angle is always near to 3 degrees. This is surprising in that the machines are of a large variety of sizes and running at different speeds. All of the screws are two-stage, however. Also, the barrel zone temperature was nearly the same for all machines. It appears that the solids angle is independent of dimensions and screw speed.



PET Solids Angle Single Stage and Barrier Screws

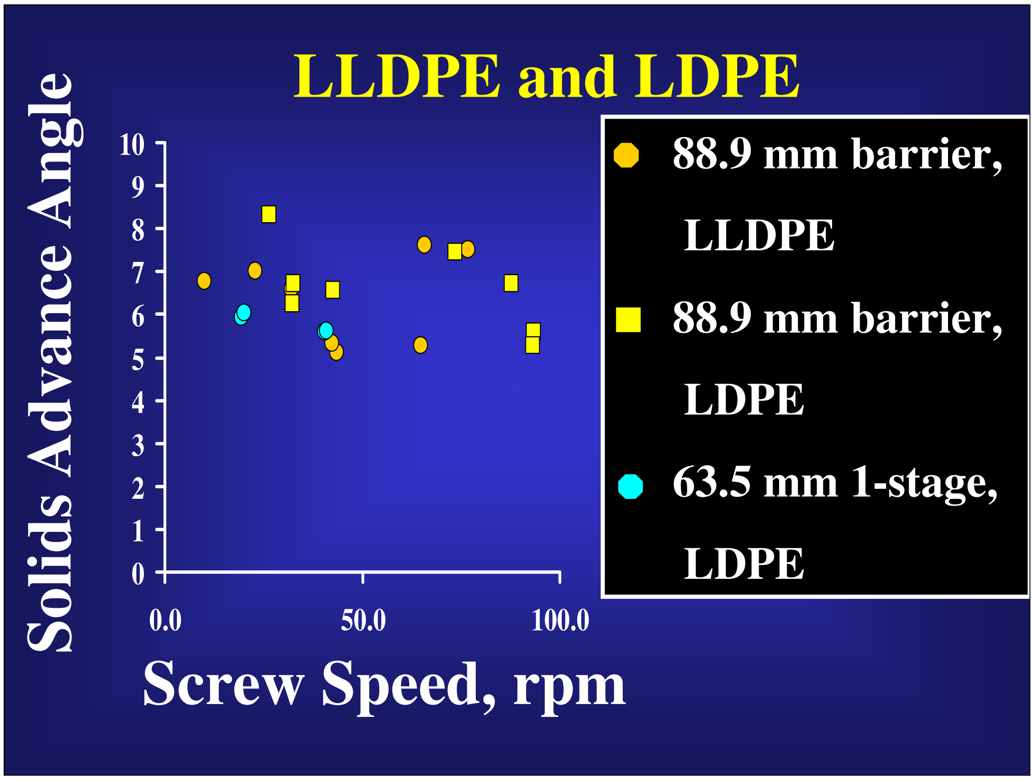
More PET data are shown for single stage barrier screws. A solids conveying angle of about 4.5 degrees prevails, except at low speeds. PET pellets and powder exhibit the same behavior.

Likely, the low speed data are for low pressure. This will be born out later.

PE DATA

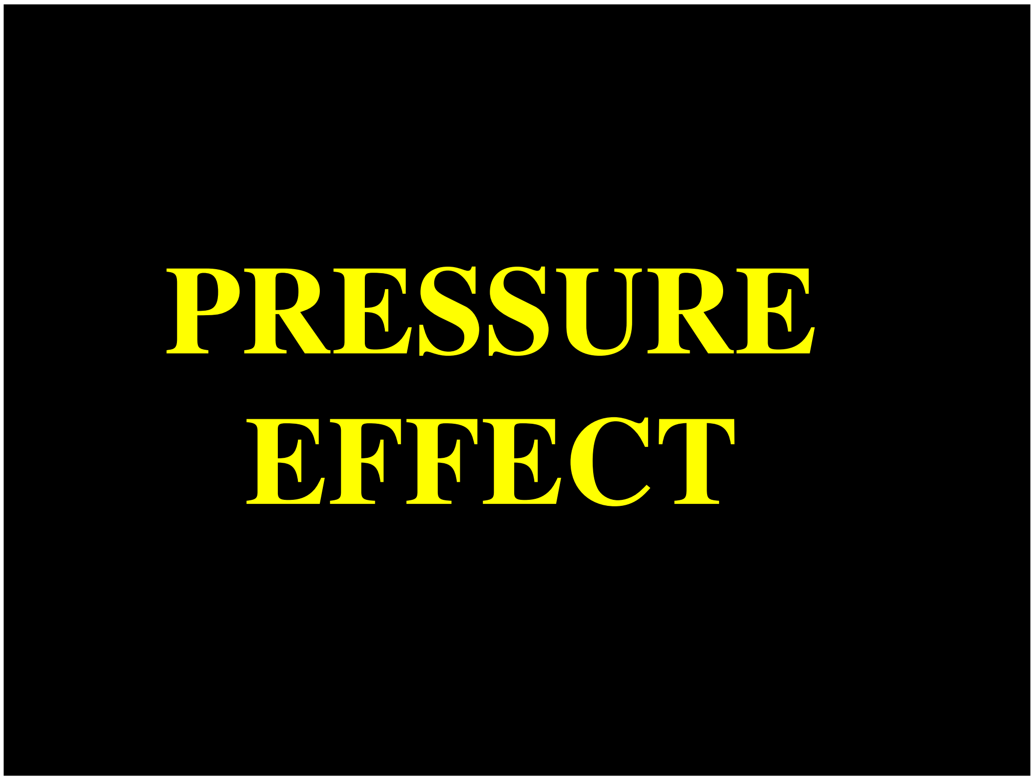
Castillo, R. J., et al., ANTEC 2002

Now, some similar data for PE resin.



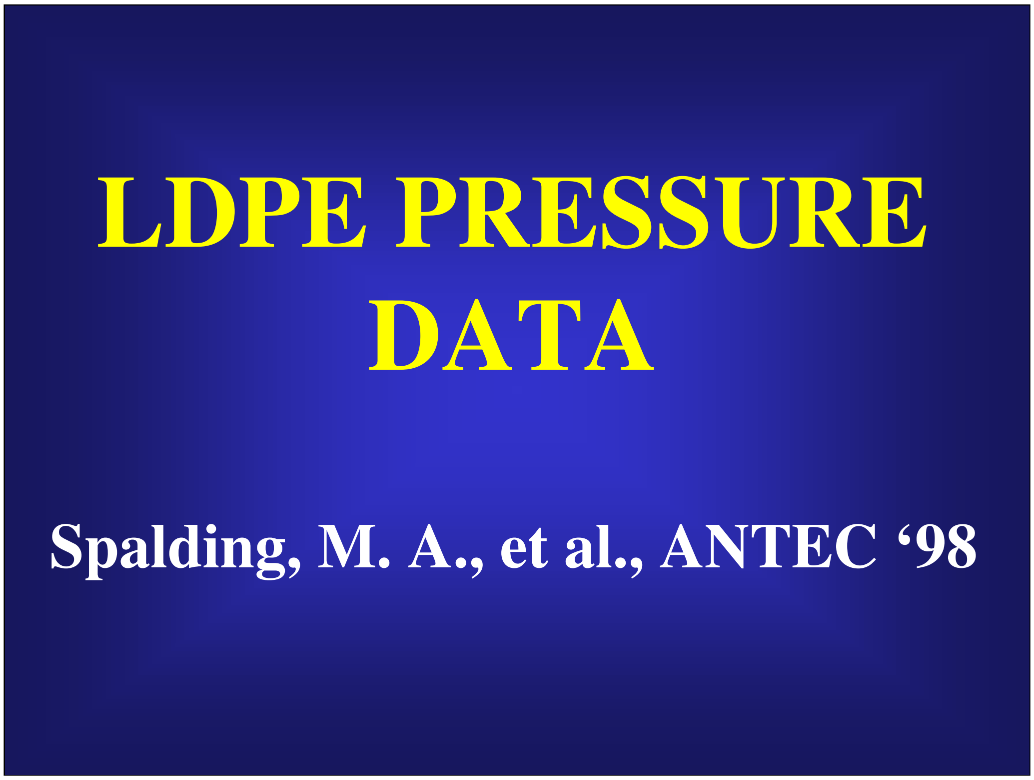
LLDPE and LDPE

The PE resin data demonstrate more scatter that did the PET resin data. This could be a result of different barrel and screw temperatures and different pressures. These factors were not included in the reference.



PRESSURE EFFECT

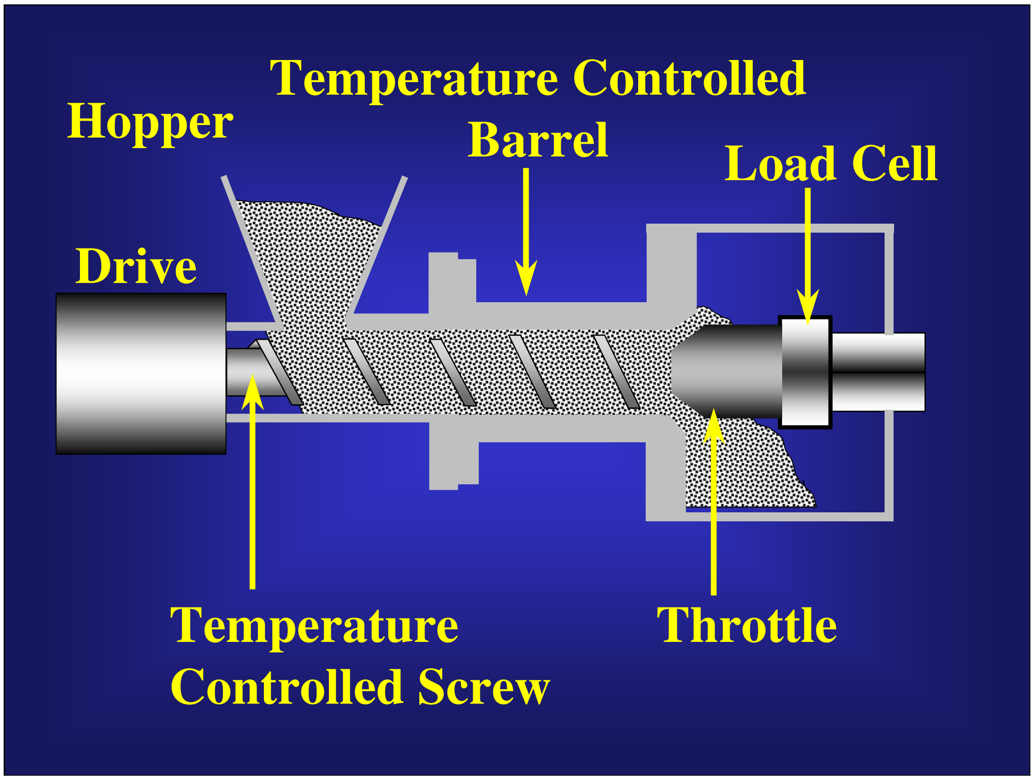
The effect of pressure on the solids conveying angle is needed.



LDPE PRESSURE DATA

Spalding, M. A., et al., ANTEC ‘98

Flow versus pressure data for solids conveying of LDPE was presented in the reference.



Temperature Controlled Barrel

Hopper

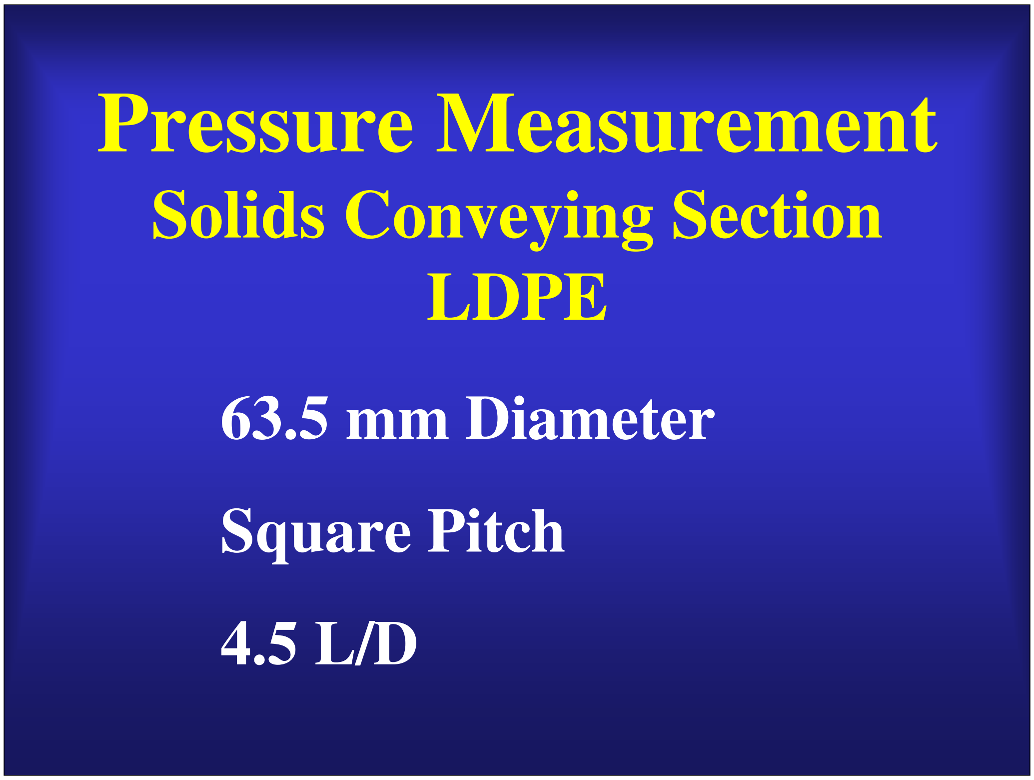
Load Cell

Drive

Temperature Controlled Screw

Throttle

A special solids conveying section was build to accurately measure the solids flow versus pressure for the LDPE. Temperatures were also carefully controlled and reported.



Pressure Measurement

Solids Conveying Section

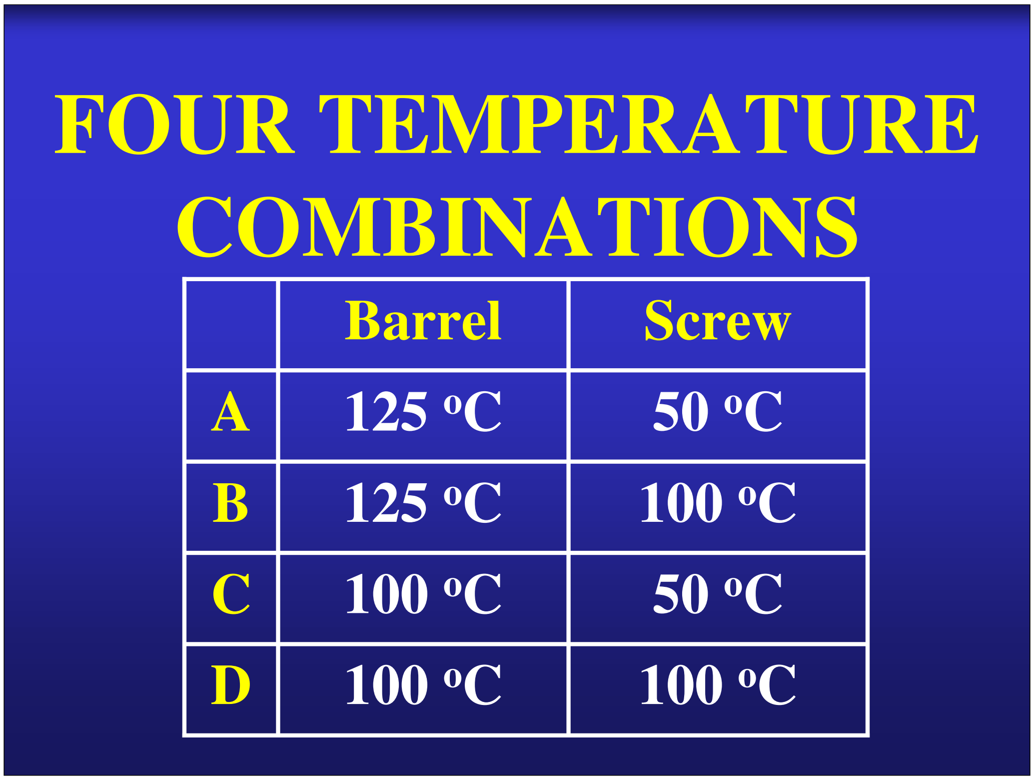
LDPE

63.5 mm Diameter

Square Pitch

4.5 L/D

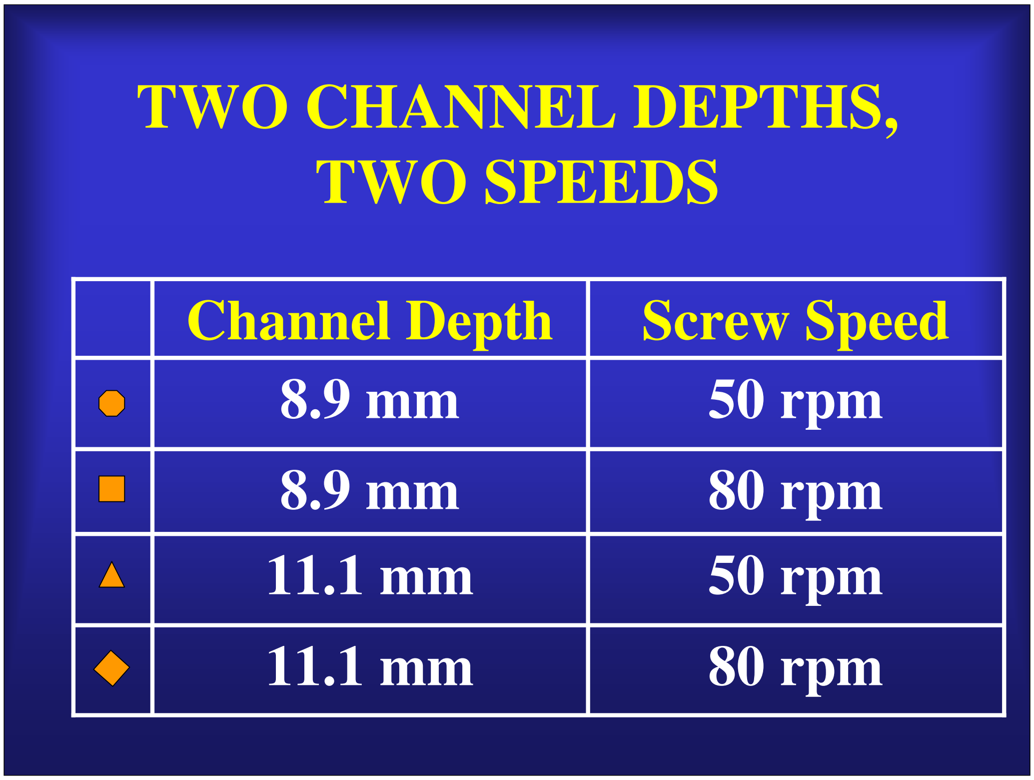
Here are the basic dimensions of the device.



FOUR TEMPERATURE COMBINATIONS

	Barrel	Screw
A	125 ^{o}C	50 ^{o}C
B	125 ^{o}C	100 ^{o}C
C	100 ^{o}C	50 ^{o}C
D	100 ^{o}C	100 ^{o}C

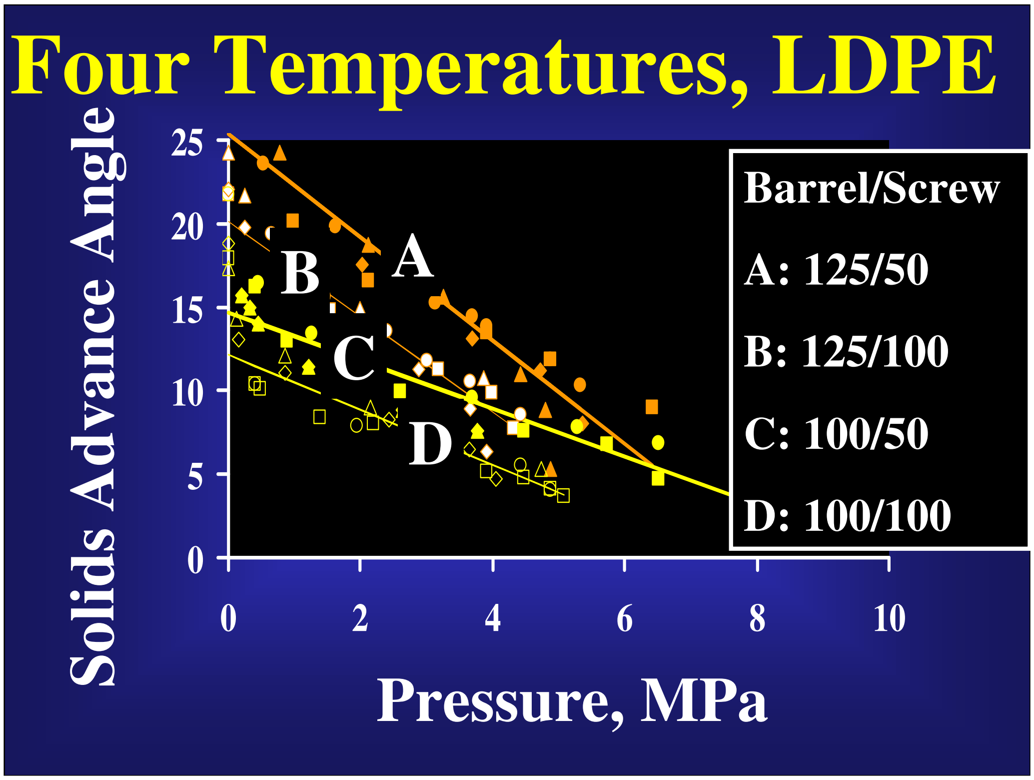
Data were taken for four combinations of barrel and screw temperature.



TWO CHANNEL DEPTHS, TWO SPEEDS

	Channel Depth	Screw Speed
●	8.9 mm	50 rpm
■	8.9 mm	80 rpm
▲	11.1 mm	50 rpm
◆	11.1 mm	80 rpm

Two different screws of different channel depths were also used. Notice the symbols on the left to designate the data to be plotted for the different screws at two different speeds.



Four Temperatures, LDPE

The data so obtained were used to calculate the solids conveying angle and plot it versus pressure.

Again, channel depth and screw speed are not major factors in the data. This was the initial observation for the PET data taken from operating machines.

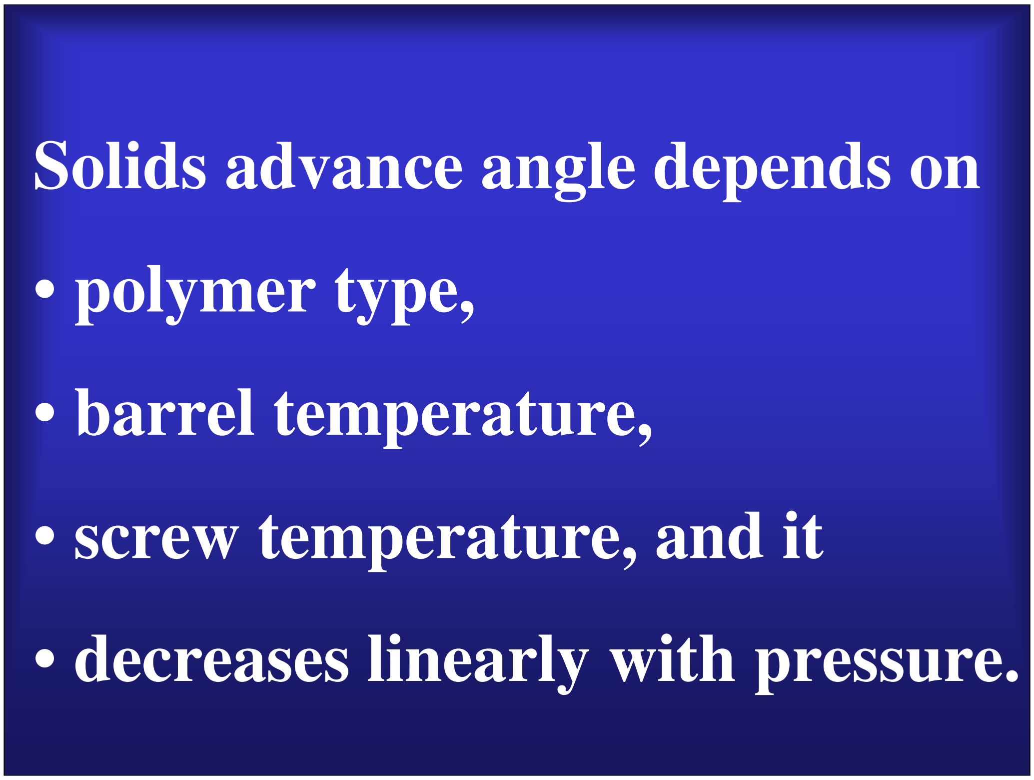
The data are linearly correlated with pressure.

Barrel and screw temperatures are the predominate factors in correlating the solids angle with pressure.

Notice how curve A and B are almost parallel. The difference is only in the screw temperature.

Curves A and C are not parallel, and they cross at a pressure of about 7 Mpa. The only difference is the barrel temperature. At 7 MPa pressure, a change in barrel temperature for this polymer would not affect the solids conveying! Below 7 MPa, raising the barrel temperature from 100 ^oC to 125 ^oC would increase the solids angle and flow. Above 7 MPa, raising the barrel temperature from 100 ^oC to 125 ^oC would lower the solids angle and flow.

Also, higher pressures are probably possible with the 100 ^oC barrel temperature if the linear approximation is accurate if extrapolated. Extrapolation of the data for A and C demonstrate that flow will stop for case A at about 8 Mpa, and flow will stop for case C at about 10 Mpa.



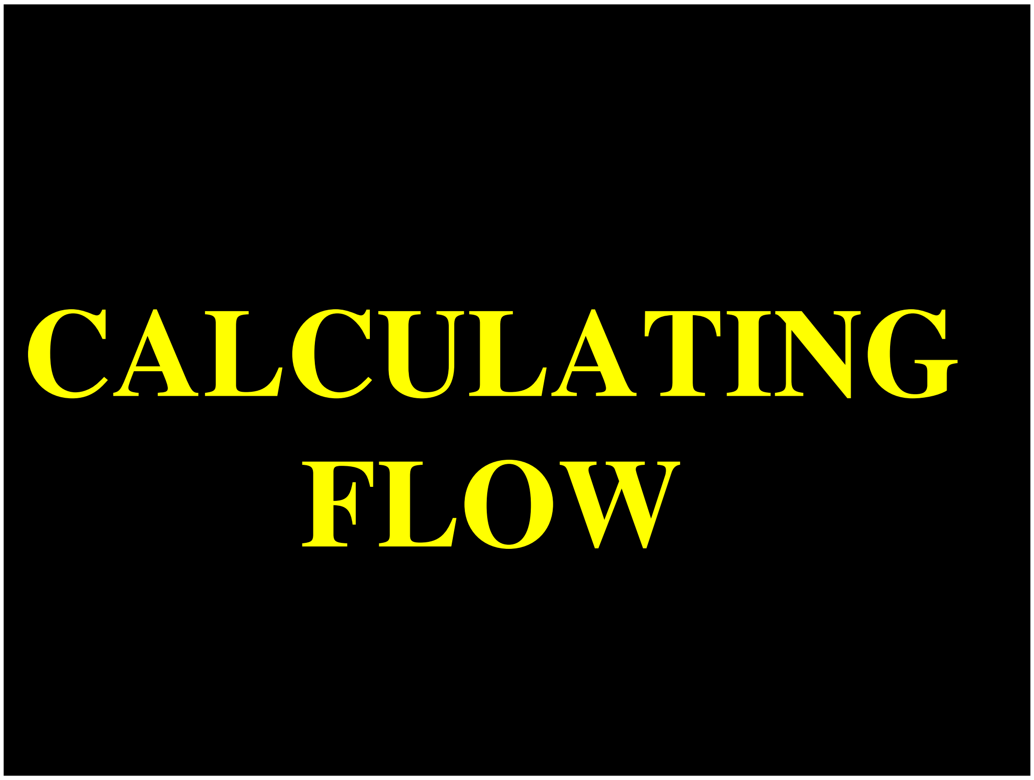
Solids advance angle depends on

- **polymer type,**
- **barrel temperature,**
- **screw temperature, and it**
- **decreases linearly with pressure.**

Therefore, solids angle depends on the above factors. Screw speed and dimensions were not major factors.

From the earlier data, it was also shown that the screw diameter was not a major factor.

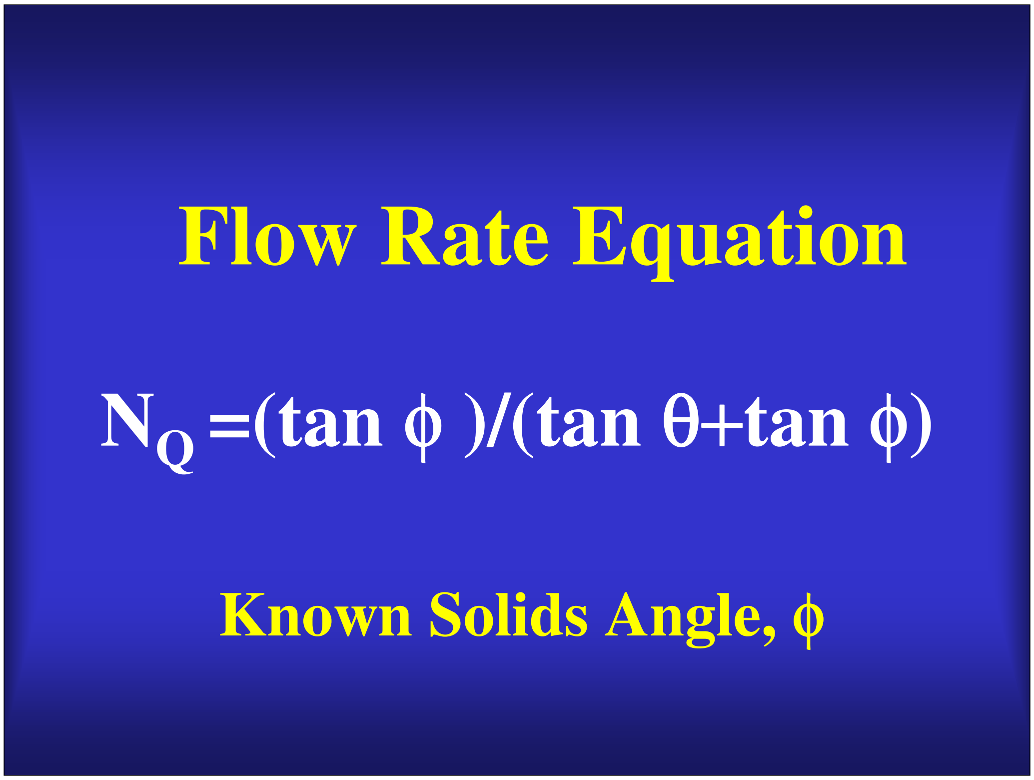
The solids conveying angle appears to be a viable design factor for solids conveying when used with the kinematics model.



CALCULATING FLOW

The solids conveying model can then be used to calculate the solids plug flow for different extruders.

The kinematics model with the solids advance angle can be used to calculate solids conveying flow.



Flow Rate Equation

$$N_Q = (\tan\phi)/(\tan\theta + \tan\phi)$$

Known Solids Angle, ϕ

Here is the basic equation for the solids flow based on the solids conveying angle. It is based on the kinematics of the solids plug using cylindrical coordinates.

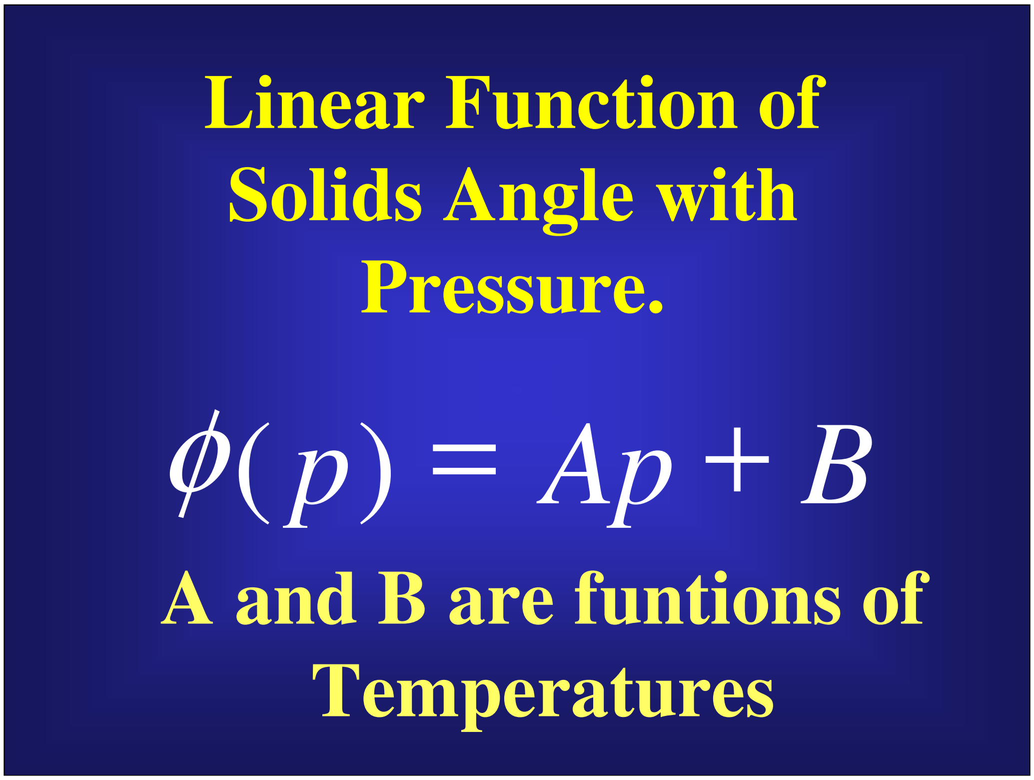
A linear function of solids angle and pressure is assumed.

In order to develop the solids flow equation to include pressure, the linear function of solids angle with pressure is used. It will have constants that depend on barrel and screw temperatures.

Different conditions on barrel and screw must be acknowledged for the solids angle. Therefore, it is construed that the effect of two different friction factors are being included in the analysis since friction factor is a major function of temperature.

The kinematics model does not disregard the fact that friction is part of the process, it just provides a method for calculating the solids flow without to knowing what are the friction factors.

The kinematics method provides a means for replacing tow independent constitutive values with a single constitutive value.

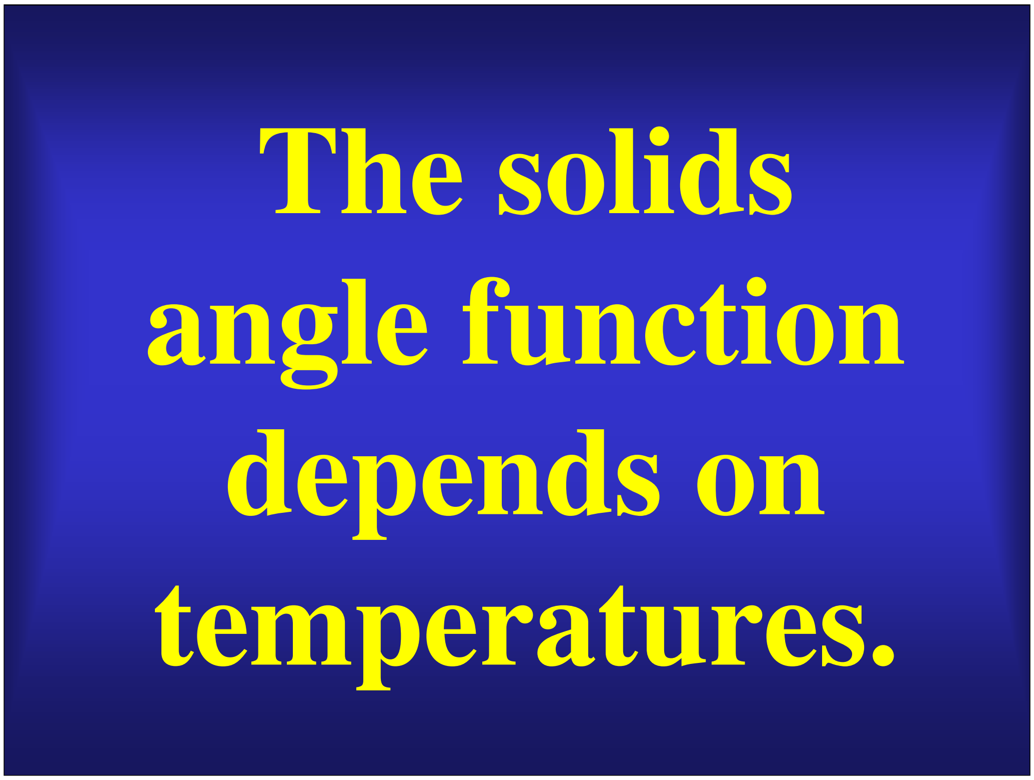


Linear Function of Solids Angle with Pressure.

$$\phi(p) = Ap + B$$

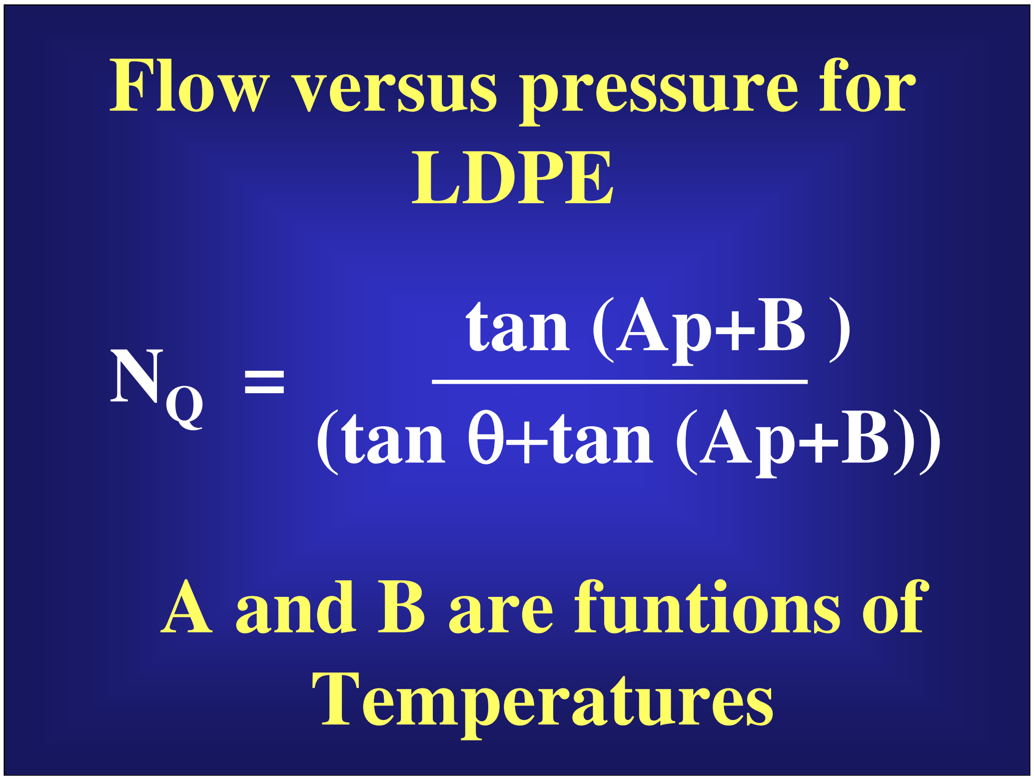
A and B are funtions of Temperatures

The effect of pressure was found to be portrayed by the above linear relationship.



The solids angle function depends on temperatures.

Values of A and B were found to be functions of barrel and screw temperatures.

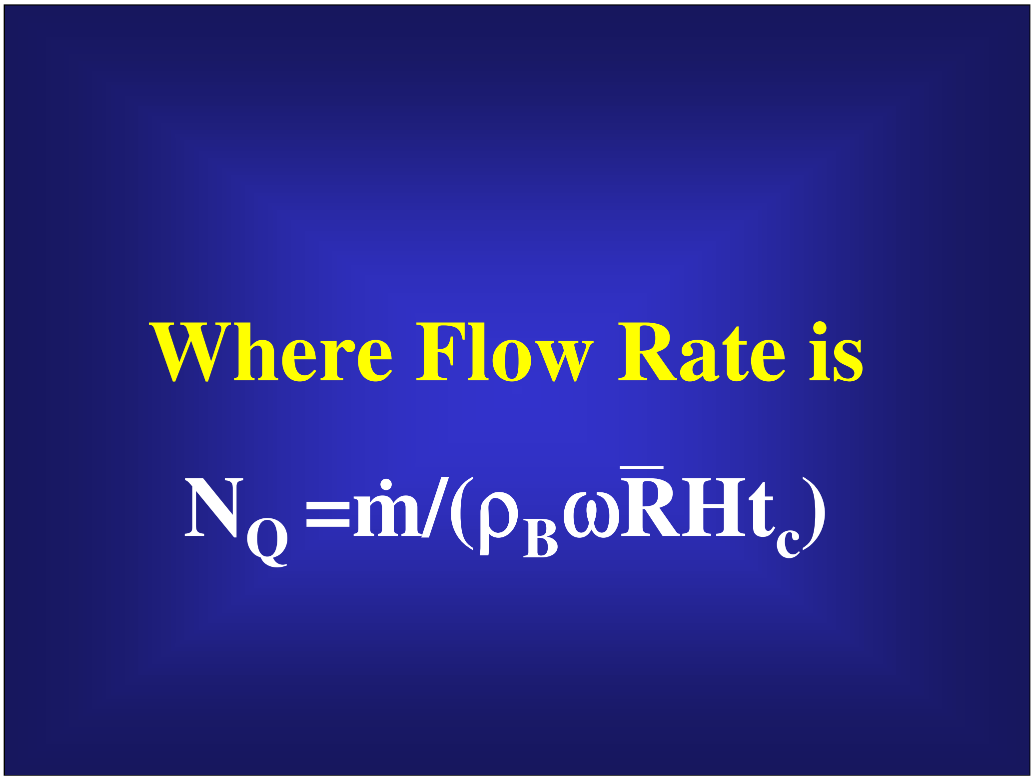


Flow versus pressure for LDPE

$$N_Q = \frac{\tan(Ap+B)}{(\tan\theta+\tan(Ap+B))}$$

A and B are funtions of Temperatures

The basic kinematics flow equation is modified to include the pressure and surface temperatures of the extruder.



Where Flow Rate is

$$N_Q = \dot{m}/(\rho_B \omega \bar{R} H t_c)$$

Dimensionless flow rate is related to actual flow rate, as shown. It is important to note that the density is a function of pressure, too.

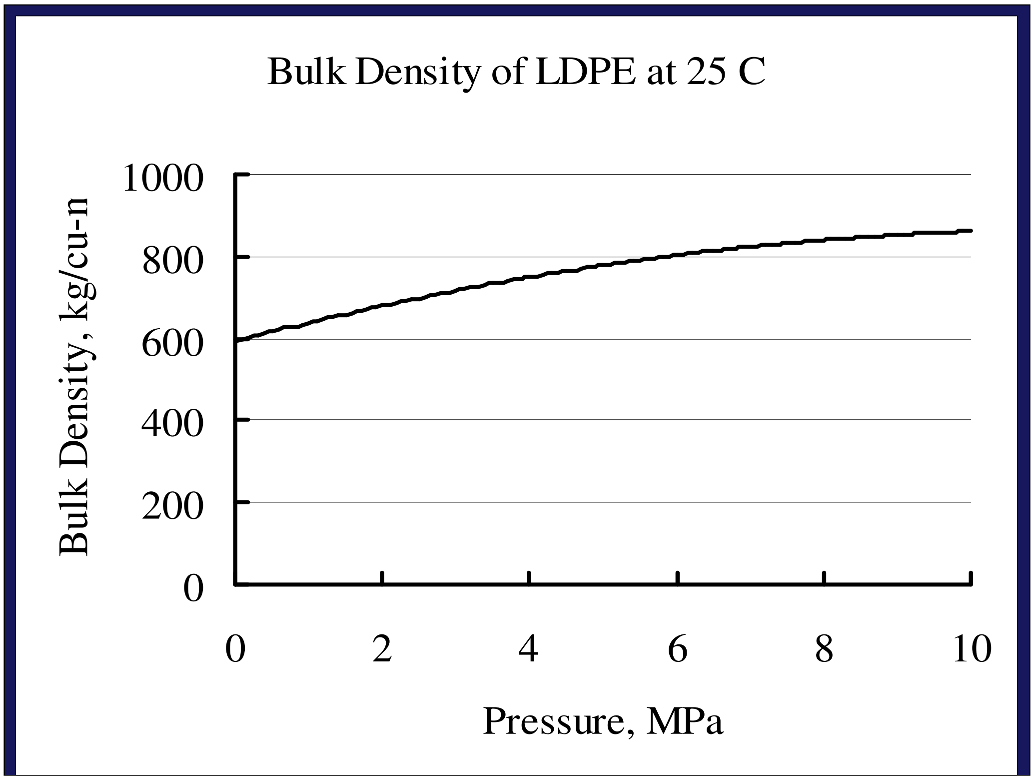
Flow versus pressure for LDPE

$$\dot{m} = \rho_B(p)\omega\bar{R}Ht_c \frac{\tan(Ap+B)}{\tan\theta + \tan(Ap+B)}$$

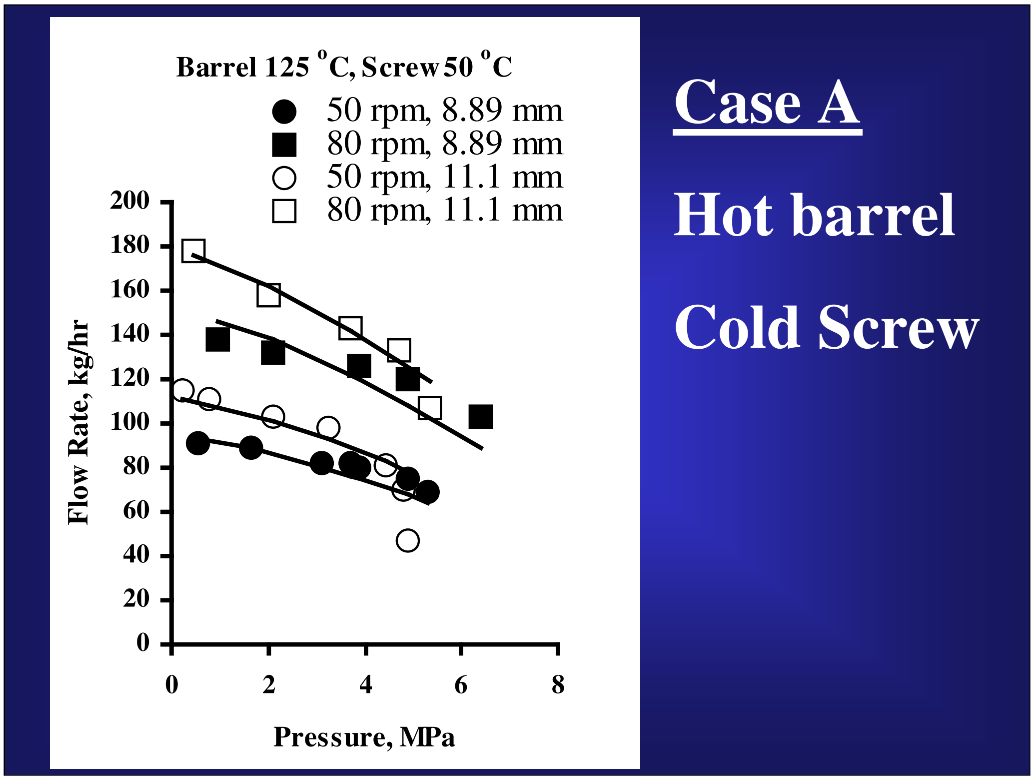
A and B are functions of Temperatures

Density, ρ_B, is a function of pressure.

The model predicts the actual flow versus pressure and surface temperatures.
Density if an important factor for each polymer resin.



Here is an example of density for LDPE. Values for many resins can be found in the literature. Mixtures and blends should be measured as a function of pressure, as shown here.



Case A

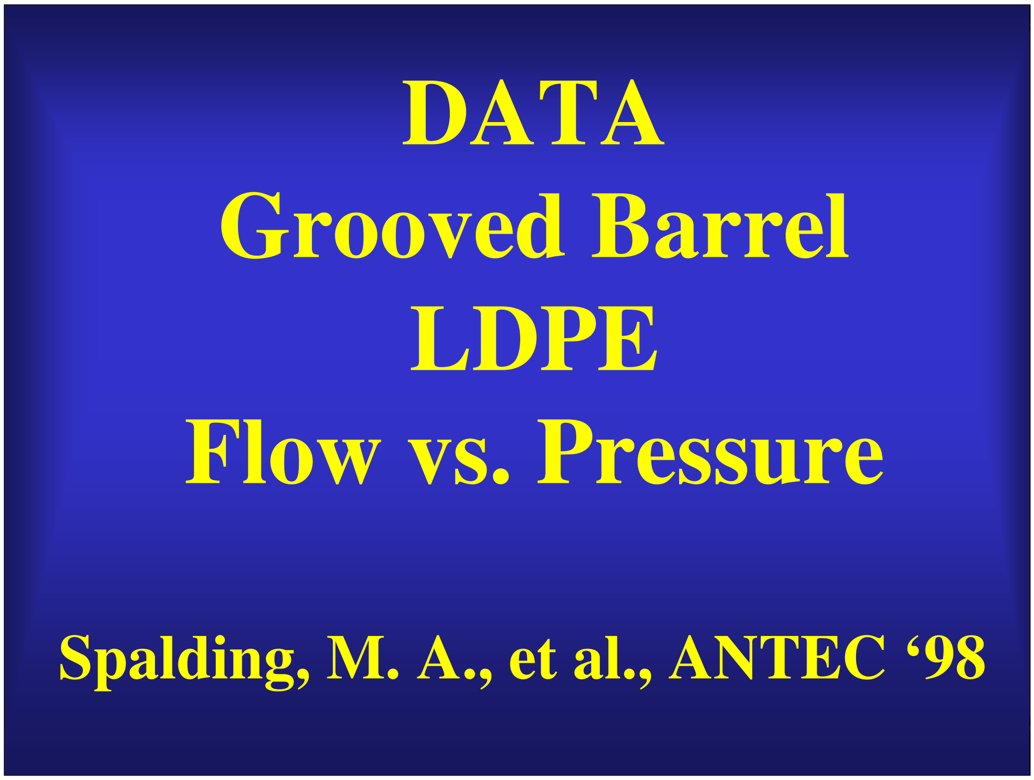
Hot barrel

Cold Screw

Here is one set of flow data for LDPE. The data are modeled by the kinematics model, and those results are the solid lines. A single barrel and screw temperature were used, as shown.

GROOVED BARREL SOLIDS FLOW

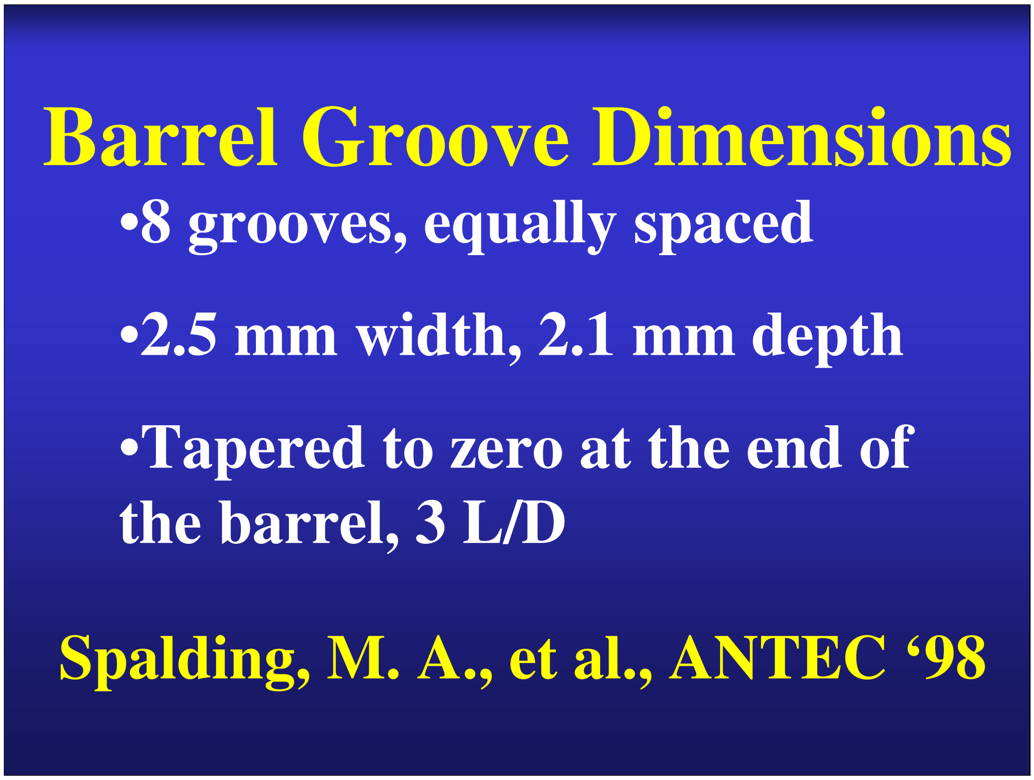
Grooved barrel solids conveying represent a unique condition. Obviously, a very difficult problem to analyze with traditional friction factors.



DATA
Grooved Barrel
LDPE
Flow vs. Pressure

Spalding, M. A., et al., ANTEC '98

Again, the test results of Spalding, et al., are used for grooved barrel perfomance. The same test facility was used as for the smooth barrel result, with just a change in the barrel.



Barrel Groove Dimensions

- 8 grooves, equally spaced
- 2.5 mm width, 2.1 mm depth
- Tapered to zero at the end of the barrel, 3 L/D

Spalding, M. A., et al., ANTEC '98

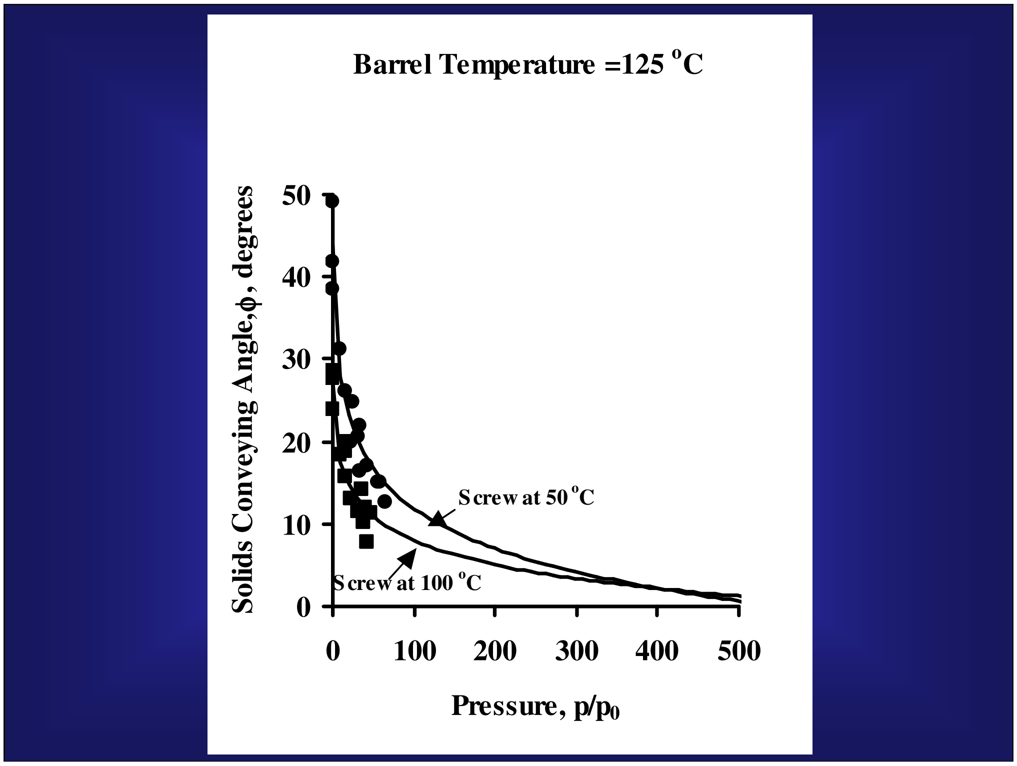
The barrel had the above grooves in it for the following data sequence.

Test Conditions

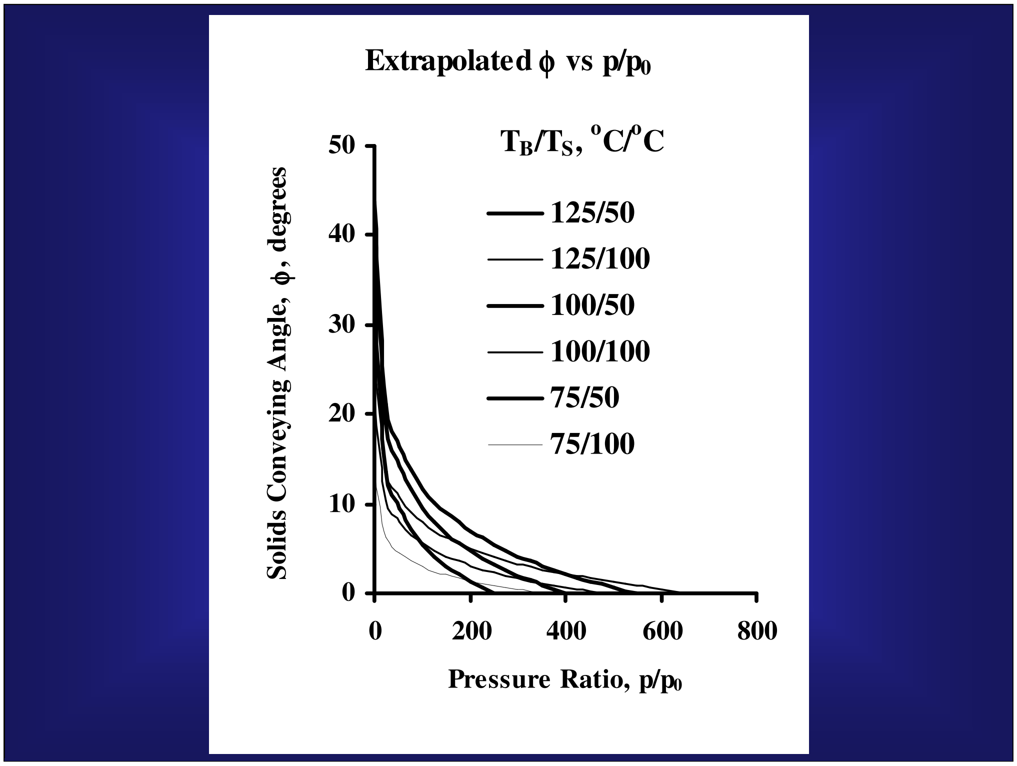
- Speeds of 50 rpm and 80 rpm
- Channel Depths of 8.89 mm and 11.1 mm
- 6 Pairs of Barrel and Screw Temperatures

Spalding, M. A., et al., ANTEC ‘98

The same screws were used as for the smooth barrel data..



Solids conveying angle for a grooved barrel versus pressure again showed correlation independent of speed and channel depth. This is typical for all temperatures tested.



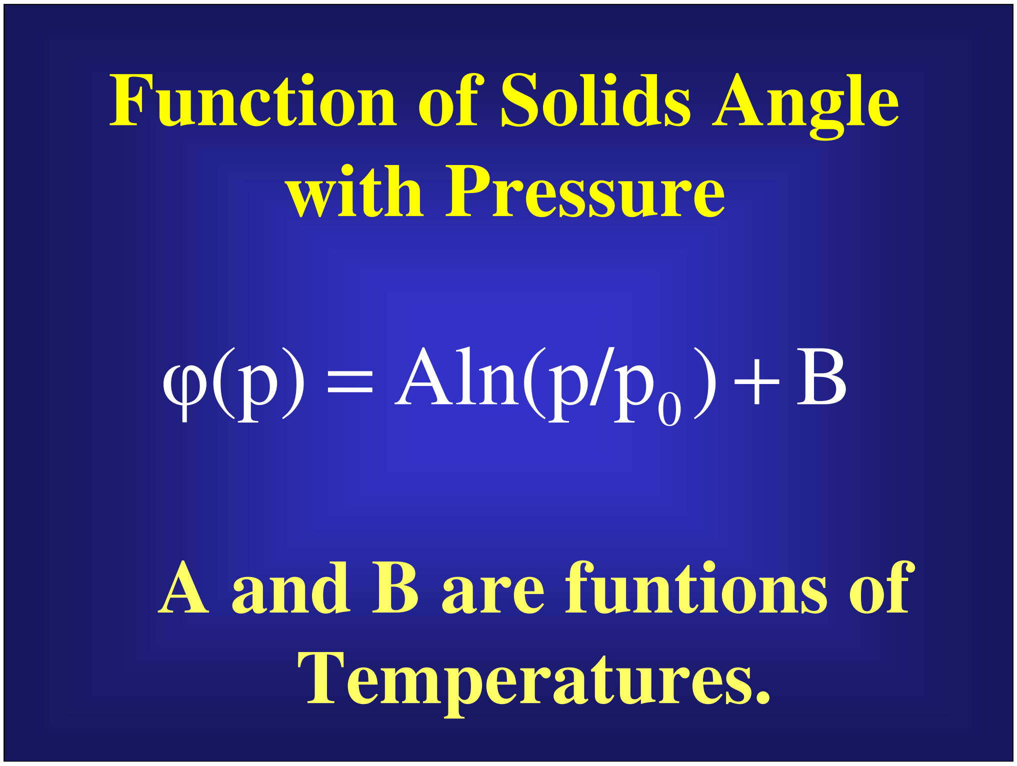
Six combination of surface temperatures were tested. A logarithmic correlation of solids angle to pressure ratio was used to model the results. The lines show that the results were very regular as temperatures were changed. Such regularity was unexpected, but it leads to confirmation that the solids conveying angle is a viable means for analyzing solids flow for grooved barrels.

The highest barrel temperature tested had the greatest solids advance angle. However, at high pressure the curves for the colder screw have greater solids angles.

A useful result of the extrapolation of the curves is that they predict the pressure point of zero flow (zero advance angle.)

Another observation is that a higher pressure is needed to stop the flow for the warmer screw temperatures, everything else being the same.

Also, the fact that the curve for low screw temperature crosses the curve for high screw temperature means that at the pressure of the cross point changes in screw temperature will not affect the flow. In a practical sense, this demonstrates the reason for seeming lack of response of solids flow performance to temperature changes in some instances.



Function of Solids Angle with Pressure

$$\varphi(p) = A\ln(p/p_0) + B$$

A and B are funtions of Temperatures.

The logarithmic function is dependent on barrel and screw temperatures. Factors A and B are a function of barrel and screw temperatures and are determined by regression analysis for each temperature pair.

So, in order to use the results, the barrel and screw temperature must be known. In other words, the barrel and screw friction factors must be set even though they are not evaluated.

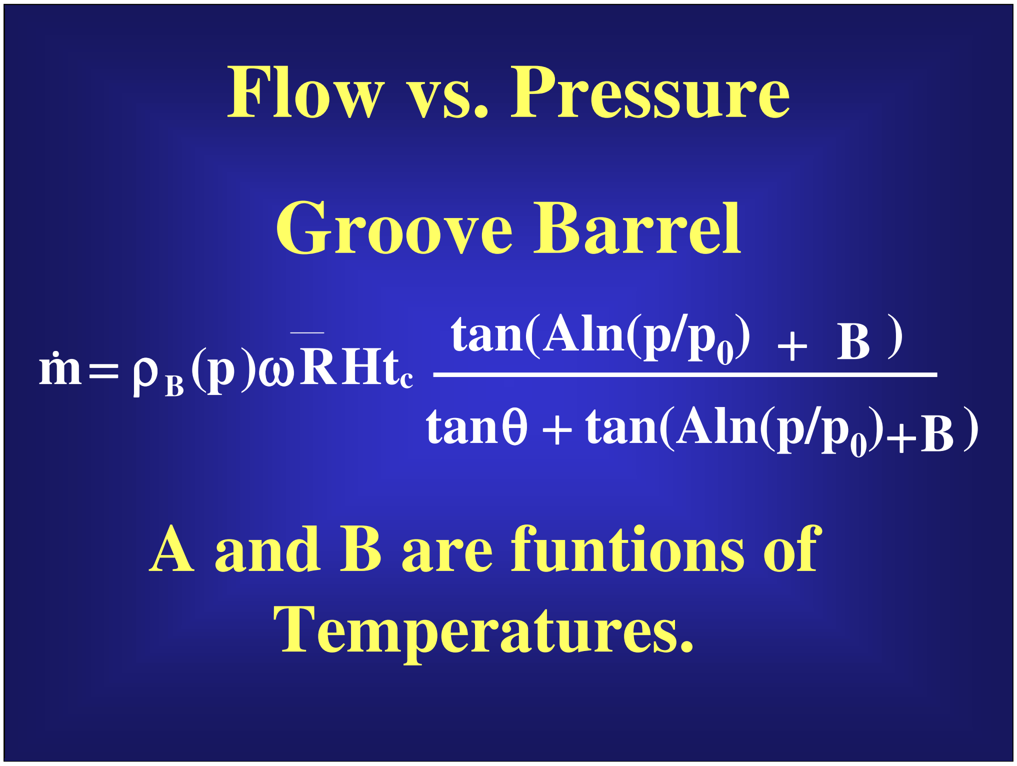
Groove Barrel Results

Solids conveying angle depends on:

- Barrel Temperature
- Screw Temperature, and
- Logarithm of the pressure ratio.

Similar to the smooth barrel results, the solids conveying angle depends on the surface temperatures and the pressure. However, a linear relationship was assumed for the angle vs pressure for the smooth barrel. The logarithmic relationship seems to more accurately fit the grooved barrel function.

GROOVED BARREL: FLOW vs PRESSURE



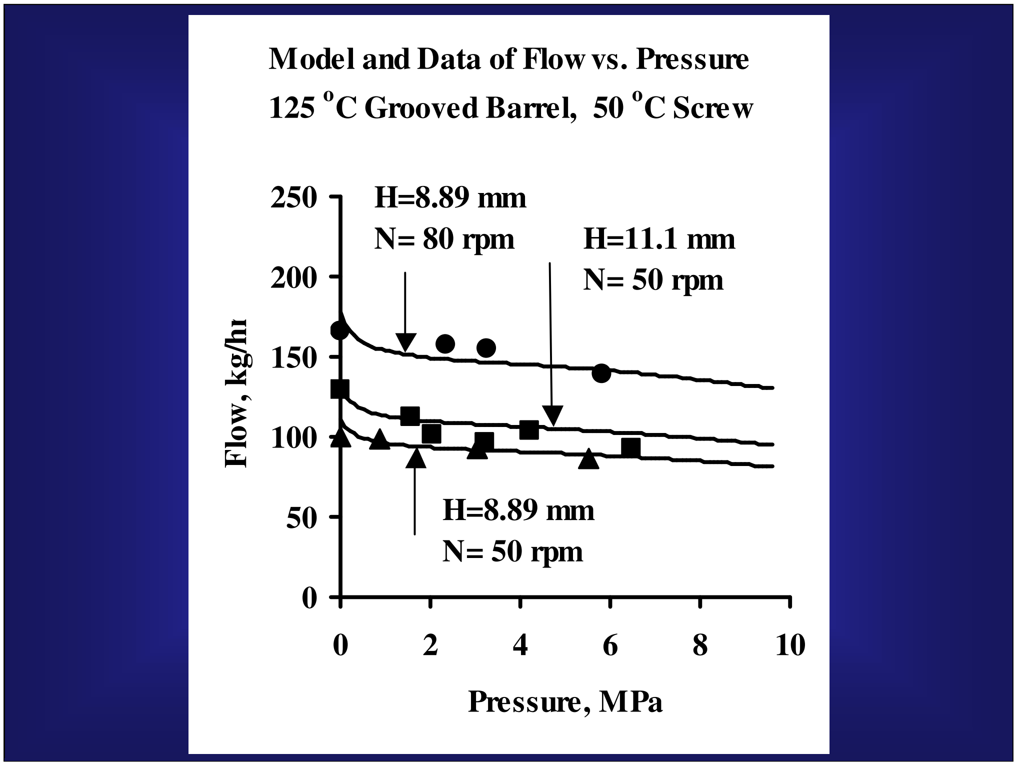
Flow vs. Pressure

Groove Barrel

$$\dot{m} = \rho_B(p)\omega\bar{R}Ht_c \frac{\tan(A\ln(p/p_0) + B)}{\tan\theta + \tan(A\ln(p/p_0)+B)}$$

A and B are funtions of Temperatures.

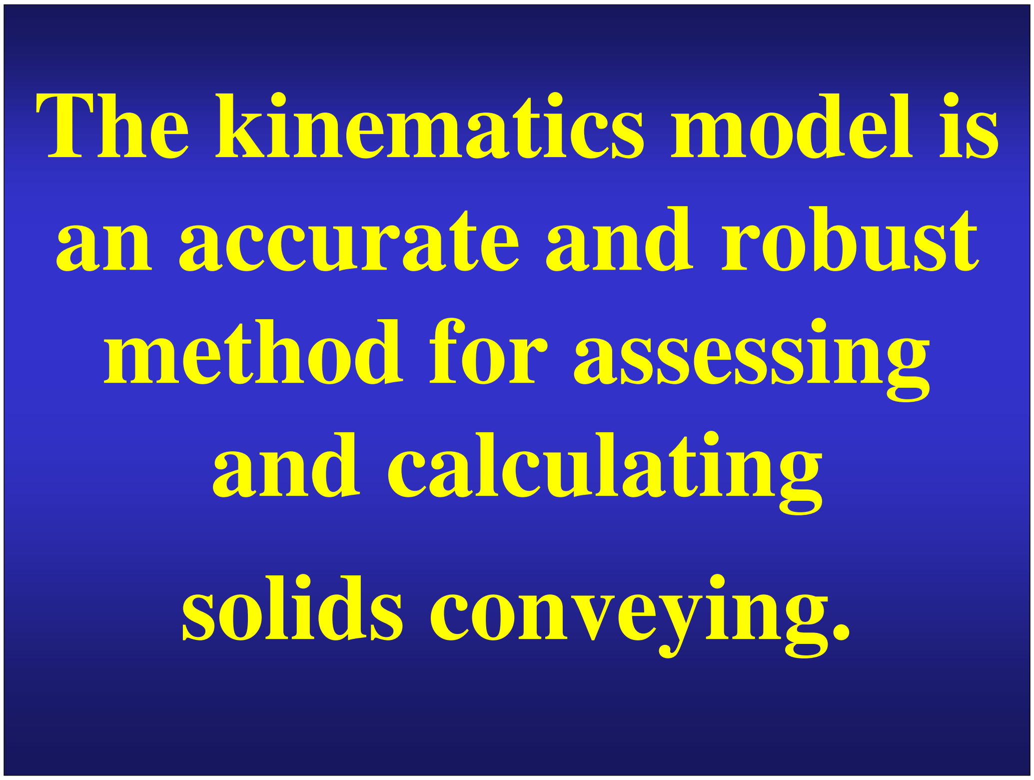
The kinematics model results in this final equation for the flow vs pressure for a grooved barrel extruder. Again, A is a function of barrel temperature, and B is a function of screw temperature. The bulk density of the polymer resin must again be supplied as a function of pressure. Ambient temperature conditions are assumed to prevail for the density as it conducts heat poorly and has a relatively short residence time in the solids conveying section.



Model and Data of Flow vs. Pressure
125 °C Grooved Barrel, 50 °C Screw

Here is a sample of the calculated flow versus pressure for the grooved barrel tests of Spalding. The points are the data, and the lines were generated by the kinematics model.

CONCLUSIONS



The kinematics model is an accurate and robust method for assessing and calculating solids conveying.

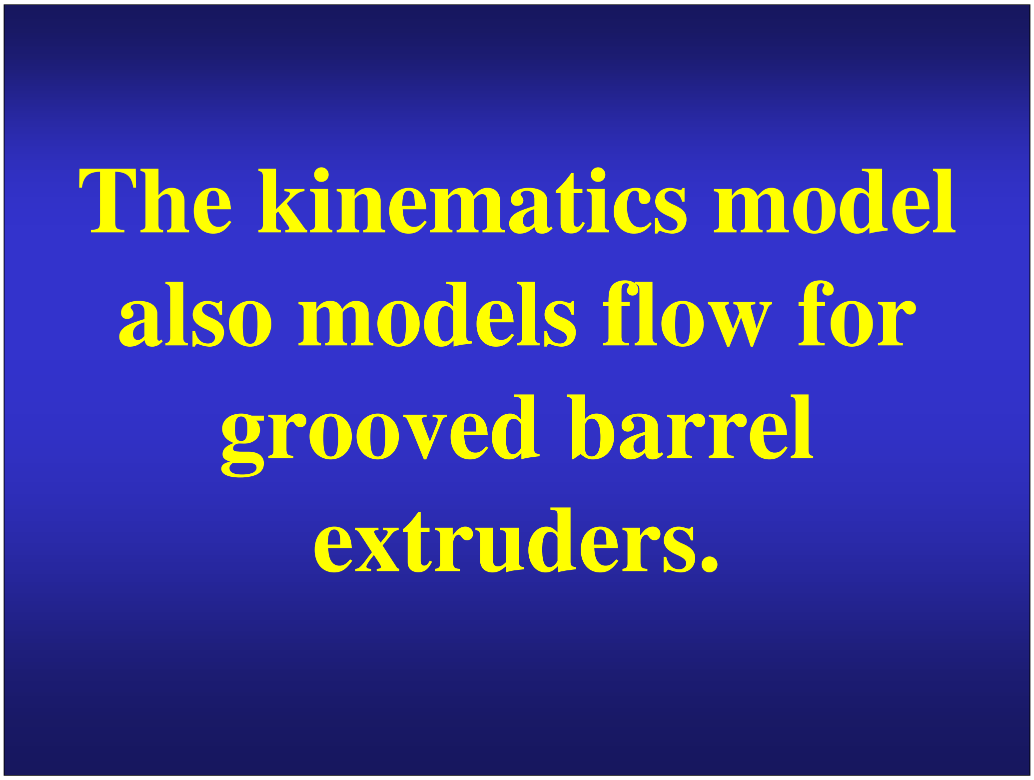
The kinematics model provides an explicit equation for the flow versus pressure of solids conveying. It also can be inverted to solve for the solids angle from the known flow of a machine. This is a very useful tool for analyzing operating machines, especially if solids pumping is suspected as causing problems.

The kinematics model does not use friction factors, which are difficult to quantify.

The kinematics model does use barrel and screw temperatures, in effect, the contribution of friction factors.

Friction factors are not used explicitly in the kinematics model, but the physics captured by the model does not neglect their existence. Put another way, exactly the same process values of barrel and screw temperatures are needed for the kinematics model as for the typical friction factor model.

The kinematics approach gives the advantage that the effect of the two friction factors (barrel and screw) can be represented by a single constitutive value (solids advance angle.)



The kinematics model also models flow for grooved barrel extruders.

Grooved barrel results for flow versus pressure were shown to be model well with the kinematics approach.

KINEMATICS OF SOLIDS CONVEYING

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