CONTROL VOLUME ANALYSIS OF FEED FLOW IN EXTRUDERS

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Abstract

The motion of the feed in a single-screw extruder is calculated with the use of a triangular control volume to determine the feed velocity and direction. A cylindrical coordinate system with a rotating screw is used to obtain torque, force, and energy balances needed to determine the developing stress (pressure) and temperature of the feed plug. The resulting equations can be easily programmed for evaluation with a personal computer.

Background

The original work of Darnell and Mol [1] formulates the basic motion and force balances for solids conveying. Constant friction factors are utilized and the flow is isothermal. Tadmor and Klein [2] improved the basics with different barrel and screw friction factors. Tadmor and Broyer [3,4] introduced heat transfer and power consumption into the equations for solids conveying and were able to account for the variation in friction factor with temperature. Chung, Hennessey and Tusim [5] added the approach of independently measuring surface stress for calculating feed flow. Hyun, Spalding, and Hinton [6] utilized the surface stress technique with a new model using two torque balances. They provide flow data from a special feed section analysis extruder with controlled temperature and pressure conditions [7]. Campbell and Dontula [8] develop a model and neglect any torque balance equation by assuming the solid bed to be an elastic fluid.

The model presented here in cylindrical coordinates progressively describes the kinematics, force, torque, and energy balances to provide pressure and temperature as a function of given flow rate. The feed angle is a direct result of the kinematics based on a triangular annular control volume. Force and torque balances with surface shear-stress follow to provide the pressure for the given flow. Finally, the energy balance yields the temperatures.

Model

Kinematics. Figure 1 shows a triangular control volume used to determine the kinematics of the feed plug (velocity magnitude and direction.) The flow rate, m, of the extruder is known for a given rotational speed, ω. At any given axial position, z, the feed density, ρ, is assumed to be constant. The conservation of mass is given as

$$\mathbf{Q_F - Q_\theta - Q_z} = 0\,. \tag{1}$$

The rotational volume flows, Q_F and Q_θ, are given in terms of rotational speed and area. The axial flow, Q_z, is based on the given flow rate. For cylindrical coordinates, equation 1 becomes

$$\omega\mathbf{\overline{R}Ht_C} - \omega_\mathbf{S}\mathbf{\overline{R}Ht_C} - \dot{\mathbf{m}}\,/\,\rho = 0\,. \tag{2}$$

The axial feed velocity is given from the flow rate and the annular area of the z direction surface as in [2] as

$$\mathbf{v}_Z = \dot{m}\,/\,\rho\,/\,((2\pi\overline{R} - w_F\,/\,\sin\theta_B)H)\,. \tag{3}$$

The angle of the feed plug advance for cylindrical motion is given from the rotational speed of the feed plug obtained from equation 2 and the axial velocity of equation 3 as

$$\phi_\mathbf{B} = \mathbf{A\tan(v_z\,/\,R\omega_S)}\,. \tag{4}$$

The magnitude of the velocity of the feed relative to the barrel is then

$$\mathbf{v_B = \omega_S R\,/\cos\phi_B,\ \omega_S} \rangle 0\text{ , or} \tag{5}$$

$$\mathbf{v_B = v_Z,\ \omega_S} = 0\,. \tag{6}$$

The feed plug follows the screw channel, so that the magnitude of the velocity of the feed relative to the screw is given as

$$\mathbf{v_C = (\omega - \omega_S)R_C\,/\cos\theta_B}\,. \tag{7}$$

Force Balance. Figure 2 shows the force balance in the axial direction on an annular segment of the feed plug. Shear forces on the channel, flights, and barrel surfaces are offset by compressive stress on the feed plug segment. The force balance for the axial direction is then given as

$$\begin{aligned}&\mathbf{\sigma w\sin\theta_B H - \left[\sigma + (\partial\sigma/\partial z)\Delta z\right]w\sin\theta_B H}\\ &\mathbf{-\tau_B\sin\phi_B w\Delta z\sin\theta_B - \tau_C\sin^2\theta_B\frac{R_C}{R}w\Delta z}\\ &\mathbf{-\tau_P A_e\sin\theta_B - \tau_L A_e\sin\theta_B}\\ &\mathbf{+\sigma_P A_e\cos\theta_B - \sigma_L A_e\cos\theta_B} = 0,\end{aligned} \tag{8}$$

where

$$\mathbf{A_e = H\Delta z\sin\theta_B\overline{R}\,/\,R.} \tag{9}$$

Equations 8 and 9 are solved for the axial stress to give

$$\partial\sigma/\partial z = -\frac{\tau_B \sin\phi_B}{H} - \frac{\tau_C R_C \sin\theta_B}{HR} - \frac{\overline{R}(\tau_P + \tau_L)\sin\theta_B}{wR} + \frac{\overline{R}(\sigma_P - \sigma_L)}{wR\tan\theta_B}. \tag{10}$$

Torque Balance. Figure 3 shows the terms for the torque balance in the rotational direction as given by the following equation.

$$\begin{aligned}&\sigma H w \cos\theta_B \overline{R} - \left[\sigma + (\partial\sigma/\partial\theta)\Delta\theta\right] H w \cos\theta_B \overline{R} \\ &- \tau_B \cos\phi_B w \cos\theta_B R^2 \Delta\theta + \tau_C \cos^2\theta_B R_C^2 w \Delta\theta \\ &+ \tau_P A_e \cos\theta_B \overline{R}\Delta\theta + \tau_L A_e \cos\theta_B \overline{R}\Delta\theta \\ &+ \overline{R}\sigma_P A_e \sin\theta_B - \overline{R}\sigma_L A_e \sin\theta_B = 0,\end{aligned} \tag{11}$$

where

$$A_e = H\overline{R}\Delta\theta\cos\theta_B. \tag{12}$$

Equations 11 and 12 are solved to give the rotational component of stress as

$$\partial\sigma/\partial\theta = -\frac{R^2\tau_B\cos\phi_B}{H\overline{R}} + \frac{R_C^2\tau_C\cos\theta_B}{H\overline{R}} + \frac{\overline{R}(\tau_P + \tau_L)\cos\theta_B}{w} + \frac{\overline{R}(\sigma_P - \sigma_L)\sin\theta_B}{w}. \tag{13}$$

The incremental stress in the helical direction (assumed to be the primary stress) is then given by the combination of the rotational stress of equation 13 and axial stress of equation 10 as

$$d\sigma = \frac{\partial\sigma}{\partial\theta}d\theta + \frac{\partial\sigma}{\partial z}dz, \tag{14}$$

where the helical direction requires that

$$d\theta = -\frac{dz}{R\tan\theta_B}. \tag{15}$$

Adding the stress difference between the center of the channel and the flight to the stress approximates the stress on the "pushing" flight surface as

$$\sigma_P = (\sigma + \Delta\sigma_l)\lambda_l. \tag{16}$$

Similarly, for the leading flight

$$\sigma_L = (\sigma - \Delta\sigma_l)\lambda_l. \tag{17}$$

A lateral stress ratio, λ_l, is assumed to act on the stress toward the flights.

The stress difference between the center of the channel and the flights is approximated by

$$\Delta\sigma_l = -\frac{w}{2R}\frac{\partial\sigma}{\partial\theta}\sin\theta_B - \frac{w}{2}\frac{\partial\sigma}{\partial z}\cos\theta_B, \tag{18}$$

where the transverse "l" direction perpendicular to the "s" direction requires

$$d\theta = \frac{dz\tan\theta_B}{R}. \tag{19}$$

Energy Balance. A two-dimensional energy balance, similar to that of [4], is made to calculate the developing temperature of the solid plug. Cylindrical coordinates replace Cartesian coordinates [4] to give the temperature by

$$\rho C_p v_z \frac{\partial T}{\partial z} = k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \tag{20}$$

The initial temperature of the polymer resin is assumed to be know, and the boundary conditions at the barrel and screw channel are a sum of heat transfer out of the feed plug and through the barrel with heat generation from frictional shear at the interface of the feed plug and the surface. Combined heat-transfer coefficients are used to specify the heat to the barrel or screw instead of only conduction in the metal as in [4]. At the barrel (r = R,)

$$-k\left.\frac{\partial T}{\partial r}\right|_B - U_B(T - T_B) + \tau_B v_B = 0. \tag{21}$$

Similarly, at the screw channel surface (r = R_C,)

$$k\left.\frac{\partial T}{\partial r}\right|_C - U_C(T - T_C) + \tau_C v_C = 0. \tag{22}$$

Constitutive Relationships. The mechanics of the solid plug described above for motion, force, torque, and energy are coupled by empirical functions. Shear data or friction factors which relate the compressive stress to the shear stress between the feed and the extruder surfaces are, of course, crucial to the solution of developing stress in the feed plug.

Original work [1,2] used an average friction factor (for barrel or screw) which was assumed constant over the entire length of the feed section. Shear stress is given as

$$\tau_i = f_i\sigma. \tag{23}$$

However, more recent work [5,6] assumes that the shear stress on surface, i, is a function of stress, σ, surface temperature, T_i, and velocity, v_i. Also, the fact that the stress, σ, in the feed plug is likely nonisotropic is also considered [7], which requires a lateral stress ratio, λ. Therefore, the shear stresses between the feed plug and the extruder surfaces are

$$\tau_i = f_\tau(\lambda\sigma, T_i, v_i), \tag{24}$$

where any number of the four independent variables may be a function of axial position, z, and surface, i. Four surfaces are used for this model: 1) barrel, i = B, 2) channel, i = C, 3) pushing flight, i = P, and 4) leading flight, i = L.

For the polymer feed, values are needed for density, specific heat, and thermal conductivity. Heat-transfer coefficients between the feed plug and the barrel and screw are also needed. Polymer bulk density, ρ, is normally a function of stress, σ, and temperature, T. Since temperature is a function of radius, r, the density function of z must be averaged for the radial effect of temperature on it.

Thermal properties of specific heat and thermal conductivity are assumed to be constant for the feed plug. The effect of stress and temperature on thermal properties could be included if such data were available.

Heat-transfer coefficients between feed plug and extruder barrel or screw will account for any thermal resistance between the solid and the location of temperature measurement. Therefore, the coefficients may include surface resistance and metal conduction. They may be chosen to approximate adiabatic ($U_C = 0$) or isothermal ($U_B = \infty$) conditions, if appropriate. The heat-transfer coefficients will affect the rate of change of interfacial temperatures as well as their eventual steady temperature. They affect the flow through their inclusion in the shear stress function, equation 24.

Solution. Given the flow rate, polymer thermal properties, initial temperature, initial pressure, friction factors or stress functions, heat-transfer coefficients, screw geometry, screw speed, and surface temperatures, a Runge-Kutta numerical procedure will give the developing stress (or pressure) of the feed plug.

1) Equations 2 to 7 are used to obtain the velocities and directions of the feed plug relative to the barrel and screw channel.
2) Equations 16, 17, and 18 give the stresses on the flights.
3) Equation 24 gives the shear stresses.
4) Equation 10 provides the stress gradient in the axial direction.
5) Equation 13 provides the stress gradient in the rotational direction.
6) Equation 14 then gives the stress gradient in the helical direction based on the axial and rotational components of steps 4 and 5.
7) The stress gradient of step 6 is multiplied by the increment of axial distance for the increment in stress.
8) Equation 20 is solved for the gradient of temperature of the feed plug based on boundary conditions of equations 21 and 22.
9) The temperature gradient is multiplied by axial increment for the increment in the temperature profile.
10) Stress and temperature are updated with the increments of steps 7 and 9, and the process is then repeated from step 1 until the length of the feed section is completed.

Example

Data [7] are given for flow versus pressure of LDPE with a special 63.3 mm, 4 L/D feed section testing extruder. Barrel and screw temperature, channel depth, and speed are varied. Data for the shear stress of the polymer are also given as a function of stress, temperature, and velocity, which makes the data uniquely useful for analysis.

Kinematics. Figure 4 shows the feed angle for 50 rpm operation of the 8.89 mm channel-depth feed-section screw. The specific gravity for LDPE is given as a function of stress and temperature [6], and it has a range of 0.58 (at low stress) to 0.92 (at high stress.) Figure 4 shows the range of feed angle versus flow rate that the solid plug must have for this range of specific gravity from equation 4. Feed angles for 50 rpm data [7] are also calculated from equation 4 at extruder exit conditions. The data traverse between the low pressure specific gravity and the high pressure specific gravity as the flow rate is lowered by high back pressure.

Pressure and Flow. The simulation of the flow versus pressure requires the constitutive functions for shear stress and density that are given in [6]. The principal stress was assumed to be in the helical direction. A lateral stress ratio of 0.7, as suggested in [7], was used for the radial direction (for the channel and the barrel) and of 0.1 for the "l" direction (pushing and leading flight surfaces.) The lower lateral stress ratio for the flights is assumed in proportion to the larger stress ratio by the aspect ratio of the channel dimensions (H/w = 0.16, $\lambda_l = (0.16)(0.7) = 0.1$.)

A high screw heat-transfer coefficient of U_C=5680 W/sq-m/K was used to approximate isothermal screw temperature. For the barrel, neither adiabatic nor isothermal conditions gave satisfactory results with the model. Also, no information was found elsewhere for heat transfer in feed sections. Therefore, in lieu of information about the barrel heat-transfer coefficient, a constant value of U_B = 568 W/sq-m/K was assumed. This gave good results for all data with the model.

Figures 5 to 7 show the theoretical results with the data [7] for various conditions of temperature, speed, channel depth, and the constant barrel heat-transfer coefficient. All of the data compare well with the theory in trend or slope, but were offset by some constant. A likely cause is the lack of specific data in [7] for the shear stresses at very low compressive stress. Only data for shear stresses at 0.69 and 3.45 mPa compressive stress were provided. All data were linearly interpolated, and the data below 0.69 mPa were linearly interpolated to 0.0 mPa shear stress at 0.0 mPa compressive stress. The behavior of the shear stresses at very low stress was determined with the model to have a significant effect on the level of stress for all rates. Therefore, the assumptions of zero shear stress at zero compressive stress and linearity are likely causes for the offset between data and model.

Energy Analysis. Figure 8 shows the results for the temperature at the barrel for the single data set of Figure 5 at 50 rpm. The calculated results show the buildup of temperature with axial distance, and the effect lower rate (higher back pressure) has on increasing the temperature.

Conclusions

1) Screw geometry, flow rate, feed plug density, and speed can be used to calculate the kinematics of the feed plug in cylindrical coordinates.

2) Force, torque, and energy balances in cylindrical coordinates are shown to give a comprehensive model to calculate the developing stress and temperature in solids conveying. Some features of the model are

- surface shear stresses can be a function of compressive stress, velocity, and temperature,
- nonisotropic stresses are approximated with lateral stress ratios.
- stress against pushing flight is different from the stress against the leading flight, and
- temperature is given by the two-dimensional energy equation in cylindrical coordinates with heat-transfer coefficients.

3) The model compares well with independent data.

Nomenclature

A_e	Area, end of annular segment
C_P	Feed plug specific heat
f_i	Friction factors
H	Channel depth, $H=R-R_C$
k	Feed plug thermal conductivity
l	Transverse coordinate of helix
m	Mass flow rate
Q_F, Q_Z, Q_θ	Volume flow rates
r	Radial coordinate
$R, R_C, \overline{\mathbf{R}}$	Radii, $\overline{\mathbf{R}} = (R+R_C)/2$
s	Helical coordinate
t_C	Channel lead length
T, T_B, T_C	Temperatures
U_B, U_C	Heat-transfer coefficients
v_B, v_C, v_Z	Velocities
w, w_F	Width
z	Axial coordinate
Δ	Differential increment
θ	Rotational coordinate
θ_B	Helix angle
λ, λ_l	Lateral stress ratios
ρ	Feed plug density
σ	Feed plug stress
$\tau_B, \tau_C, \tau_L, \tau_P$	Surface shear stresses
ϕ_B	Feed plug advance angle
ω, ω_S	Rotational speeds

Subscripts

B	Barrel
C	Channel
e	End of annular segment
F	Flight
i	Surface
L	Leading flight
P	Pushing flight
S	Solid feed
Z	Axial direction
θ	Rotational direction

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