

# STABILITY OF TWO-STAGE SINGLE-SCREW EXTRUDERS

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## Abstract

In two-stage extrusion, the pumping section must be sized to produce the needed delivery pressure of the extruder. A method of calculating the channel depth so that the desired pressure is achieved and that the stability of the flow is optimized has been developed. Data for an example are given.

## Introduction

A common method for extracting vapors and gases during the extrusion process is to use a two-stage, vented extruder as shown in Figure 1. The first stage melts the polymer, and the second stage vents gasses and vapors and develops pressure. Venting is done in a devolatilization section of the second stage through a port in the barrel to a low pressure or vacuum. Between the vent and the extruder exit, a pumping section must develop pressure according to the load at the extruder exit. It must transport the melt flow and build the melt pressure from vacuum or low pressure in the devolatilization section to the required exit pressure.

The flow of polymer in the extruder has inherent instability, typically found in the feed hopper, the solids conveying section, and the melting process. These basic sources of instability exist in the early part of the first stage of the extruder. It will be shown here that it is possible to minimize their effect by proper design of the metering and pumping sections.

## Stability

Obviously, the metering section, as shown in Figure 1, in the first stage will dampen the flow of the solids conveying and melting. A shallow channel-depth,  $h_m$ , and a long metering-section,  $L_m$ , will maximize its damping. However, a shallow channel-depth will lower the flow rate and raise the melt temperature, so this option has limitations. Maximizing the metering-section length is likely the more practical option for improved damping. However, this is limited by the amount of screw length available.

The second stage of the screw, as shown in Figure 1, contains a devolatilization section and a pumping section. In the devolatilization section of the screw, where vapors are released, a free surface must exist on the polymer so that the gases may escape and travel to the vent port in the barrel. This free surface requires that the screw channel

be partially filled, and the free surface will continue into the pumping section. At some axial point, the free surface ends, and the screw channels in the pumping section are completely full of polymer.

An optimum pumping-section is defined as one that builds pressure,  $p_e$ , in the shortest filled axial-length,  $L_f$ , of pumping section. Theory and practice have shown that the optimum pumping-section for a two-stage extruder has a channel depth,  $h_p$  that is greater than that of the metering section,  $h_m$ , in the first stage of the extruder. For a square-pitched screw, the optimum ratio of pumping-section channel depth to metering-section channel depth,  $h_p/h_m$ , is  $3/2$  [1]. Subsequent vented-extruder designs [2-5] have specifically stated and followed this optimum channel-depth ratio, but none [2-5] address the issue of its effect on pressure stability.

However, the pumping section provides another place where stabilizing the output pressure can be greatly assisted. Output flow rate,  $Q_p$ , is assumed constant. As the metering flow,  $Q_m$ , enters the pumping section, its variation will cause the inventory of polymer in the pumping section to vary. The variation of inventory is accommodated by a change in the filled length,  $L_f$ , of the pumping section, and the filled length is in proportion to the exit pressure,  $p_e$ . If the filled length is short, then the exit pressure will be low. As the length is increased, the pressure will become higher. Therefore, the exit pressure varies according to the variation of the filled length. At constant metered exit flow rate, exit pressure instability in proportion to filled-length variability of the pumping section is the result.

## Math Model

The phenomenon of pressure stability for a two-stage extruder is now calculated with the classic one-dimensional flow model [1]. Figure 1 shows the two-stage extruder setup with first-stage metering and second-stage pumping. The volume flow-rate,  $Q_i$ , of the metering and pumping (subscript  $i = m$  or  $p$ ) for constant melt density is given [1] as

$$Q_i = \alpha_i N - \frac{\beta_i \Delta p_i}{\mu_i L_i} \quad (1)$$

The factor,  $\alpha$ , is a geometrical drag-flow factor, and it is proportional to channel depth. The factor,  $\beta$ , is a geometrical pressure-flow factor, and it depends on the cube of channel depth. The screw speed is  $N$ , the melt viscosity is  $\mu$ , and the axial pressure-difference is  $\Delta p$ .

### Pressure Stability:

The exit pressure of the extruder is the pressure of the filled length of the pumping section because the pressure in the devolatilization section is made to be a vacuum or relatively very low. Therefore,

$$p_e = \Delta p_p. \quad (2)$$

Equations 1 and 2 are combined to solve for the exit pressure of the extruder as

$$p_e = N \frac{\alpha_p \mu_p L_f}{\beta_p} - \dot{Q}_p \frac{\mu_p L_f}{\beta_p}. \quad (3)$$

Assuming the exit flow,  $Q_p$ , is held constant by a melt or metering pump, the change in exit pressure is found by differentiating Equation 3.

$$\frac{dp_e}{dt} = (N\alpha_p - \dot{Q}_p) \frac{\mu_p}{\beta_p} \frac{dL_f}{dt}. \quad (4)$$

The change in filled length,  $L_f$ , of the pumping section is related to the change in the total filled volume (inventory),  $V_f$ , of the pumping section by

$$\frac{dL_f}{dt} = \frac{dV_f}{A_p dt}. \quad (5)$$

where  $A_p$  is the cross-sectional area of the pumping channel.

When instability causes the first-stage metering flow-rate,  $Q_m$ , to differ from the extruder exit-flow,  $Q_p$ , the change in filled volume of the pumping section is given by

$$\frac{dV_f}{dt} = \dot{Q}_m - \dot{Q}_p. \quad (6)$$

The pressure of the first stage metering,  $\Delta p_m$ , is assumed to have an arbitrary incremental change,  $\delta \Delta p_m$ , which results from instability. The difference in flow of Equation 6 can be represented by

$$\begin{aligned} \dot{Q}_m - \dot{Q}_p &= \dot{Q}_p + \frac{d\dot{Q}_m}{d\Delta p_m} \delta \Delta p_m - \dot{Q}_p \\ &= \frac{d\dot{Q}_m}{d\Delta p_m} \delta \Delta p_m. \end{aligned} \quad (7)$$

The flow rate of the first-stage metering ( $i = m$ ) given by Equation 1 is differentiated to give

$$\frac{d\dot{Q}_m}{d\Delta p_m} = -\frac{\beta_m}{\mu_m L_m}. \quad (8)$$

Equation 4 is then combined with Equations 5-8 to give the rate of change of exit pressure as a function of an incremental change in pressure of the first stage. The result is

$$\frac{dp_e}{dt} = -(N\alpha_p - \dot{Q}_p) \frac{\mu_p}{\beta_p A_p} \frac{\beta_m}{\mu_m L_m} \delta \Delta p_m. \quad (9)$$

Equation 9 shows the rate of change of the exit pressure of the extruder to depend on the design factors of pressure flow,  $\beta_p$  and  $\beta_m$ , pumping-section channel-area,  $A_p$ , and metering section length,  $L_m$ .

In order to make the rate of change of pressure only dependent on screw design factors, Equation 9 is normalized by the rate of change at a channel-depth ratio of 3/2 to give stability,  $N_s$ , as

$$N_s = \frac{(dp_e / dt)}{(dp_e / dt)_{3/2}} = \frac{L_{3/2} (\beta_p A_p)_{3/2}}{L_m \beta_p A_p}. \quad (10)$$

Because the pressure-flow factor,  $\beta_p$ , is proportional to the cube of the channel depth, the area,  $A_p$ , is proportional to the channel depth, and channel-depth ratio is defined as  $h_p/h_m$ , Equation 10 is given as

$$N_s = \frac{L_{m_{3/2}} [(h_p / h_m)^3 h_m^3 (h_p / h_m) h_m]_{3/2}}{L_m (h_p / h_m)^3 h_m^3 (h_p / h_m) h_m}. \quad (11)$$

The channel-depth ratio of 3/2 yields

$$N_s = \left[ \frac{L_{m_{3/2}}}{L_m} \right] \left[ \frac{(3/2)^4}{(h_p / h_m)^4} \right]. \quad (12)$$

Equation 12 gives the damping or ratio of the pressure variation at the extruder exit to that at the inlet to the metering section of the first stage. The linear dependence of the stability of the exit pressure on the metering length is bracketed independently of the factor for the pumping-channel depth-ratio. The bracketed factor, which contains the channel-depth ratio, is plotted as percentage versus channel-depth ratio,  $h_p/h_m$ , in Figure 2.

### Pumping Pressure:

The pressure gradient of the filled length of the pumping section,  $(\Delta p/L)_f$ , is obtained from Equation 1 by equating the flow rate for the metering section to that of the pumping section and assuming that the metering pressure term is negligible. Solving for pressure gradient:

$$\left( \frac{\Delta p}{L} \right)_f = \frac{N(\alpha_p - \alpha_m) \mu_p}{\beta_p}. \quad (13)$$

The pressure gradient of Equation 13 is normalized, similarly to Equation 11, with the pressure gradient evaluated at a channel-depth ratio of 3/2 to give

$$N_p = \frac{(\Delta p / L)_f}{[(\Delta p / L)_f]_{3/2}}. \quad (14)$$

Again, with  $\alpha_i \sim h_i$ ,  $\beta_i \sim h_i^3$  [1], and Equation 13, Equation 14 is given only in terms of channel-depth ratio as

$$\begin{aligned} N_p &= \frac{(h_p / h_m - 1) (3/2)^3}{(3/2 - 1) (h_p / h_m)^3} \\ &= \frac{27 (h_p / h_m - 1)}{4 (h_p / h_m)^3}. \end{aligned} \quad (15)$$

Figure 2 shows that the pressure gradient,  $N_p$ , is not drastically reduced as the channel-depth ratio is increased above a value of 3/2. This important point means that the channel-depth ratio can be increased to gain stability without sacrificing much pressure gradient. Equation 15 as shown in Figure 2 provides the quantitative results needed to calculate larger channel-depth ratios for stability gains with acceptable moderate loss of pumping-pressure gradient.

### Example

A two-stage screw is operated that contains a channel-depth ratio of 1.87. This is shown in Figure 2 to provide a pressure variability of 41%. The screw was redesigned to have a channel-depth ratio of 2.26 and a metering-section length 1.07 x original metering-section length. Figure 2 shows that the exit-pressure variability, because of the pumping section change, decreases to 19%. The ratio in pressure variation of new design to old design is the ratio of the results of Equation 12 as follows:

$$R = \frac{[N_s]_{2.26}}{[N_s]_{1.87}} = \frac{1}{1.07} \frac{19}{41} = 0.43. \quad (16)$$

Figure 3 shows the measured data for the dynamic pressure of the old screw and the new screw. Conditions for operation were identical, and the average pressure is 1700 kPa. The pressure variability is about 450 kPa for the original design and it is reduced to about 200 kPa for the new design. The ratio of the pressure variabilities,  $R$ , is  $200/450 = 0.44$ . This is close to the value predicted by Equation 16.

Figure 2 also predicts that the exit pressure gradient is reduced from 90% to 72%. This means the pumping section length of *fill* needed to be 18% longer for the redesigned screw to provide the same average exit pressure of 1700 kPa. However, since the filled length is probably only about one-half of the total pumping-section

length, the increase in screw length is reduced by one-half to 9%. This was easily accommodated without extra pumping-section length added to the design.

The design was also tested with elaborate computer models for extrusion, and the results were the same.

### Conclusions

1. Increasing the metering section length of the first stage of a two-stage extruder screw improves output pressure stability in proportion to the inverse of the metering section length.
2. The pressure stability is improved in proportion to the inverse of the fourth power of the ratio of pumping-channel depth of the second stage to metering-channel depth of the first stage.

### References

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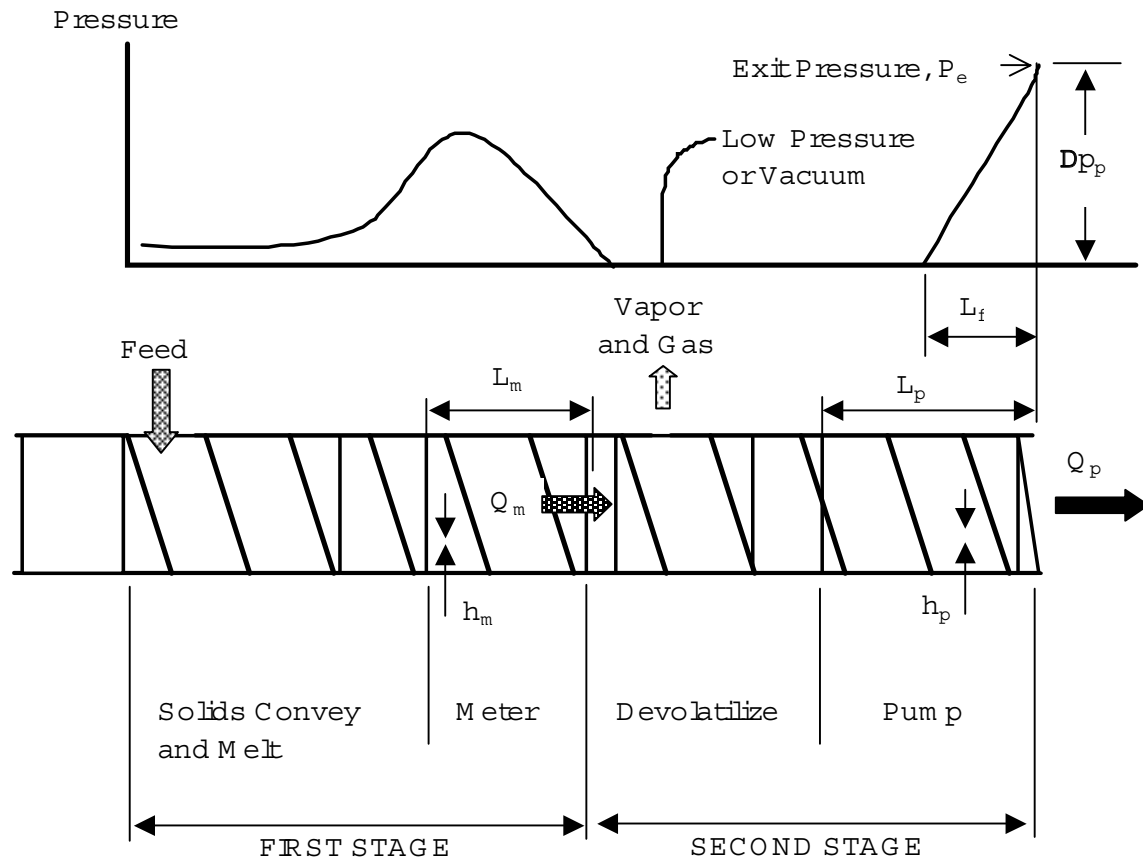


Figure 1. The two stages of a two-stage extruder screw .

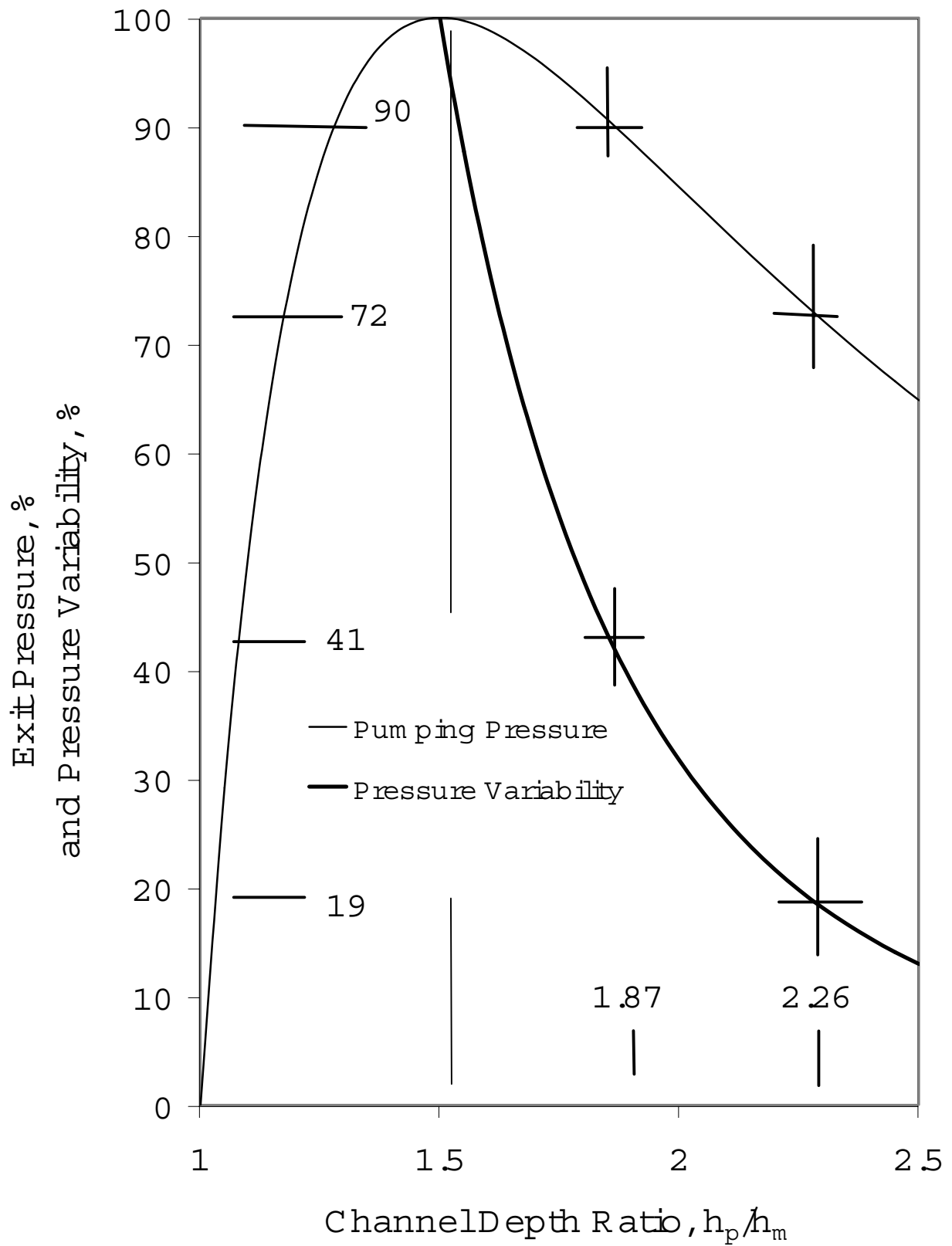


Figure 2. Exit pressure and pressure variability for square-pitched two-stage extruder screw. Model results.

### Transient Exit Pressure

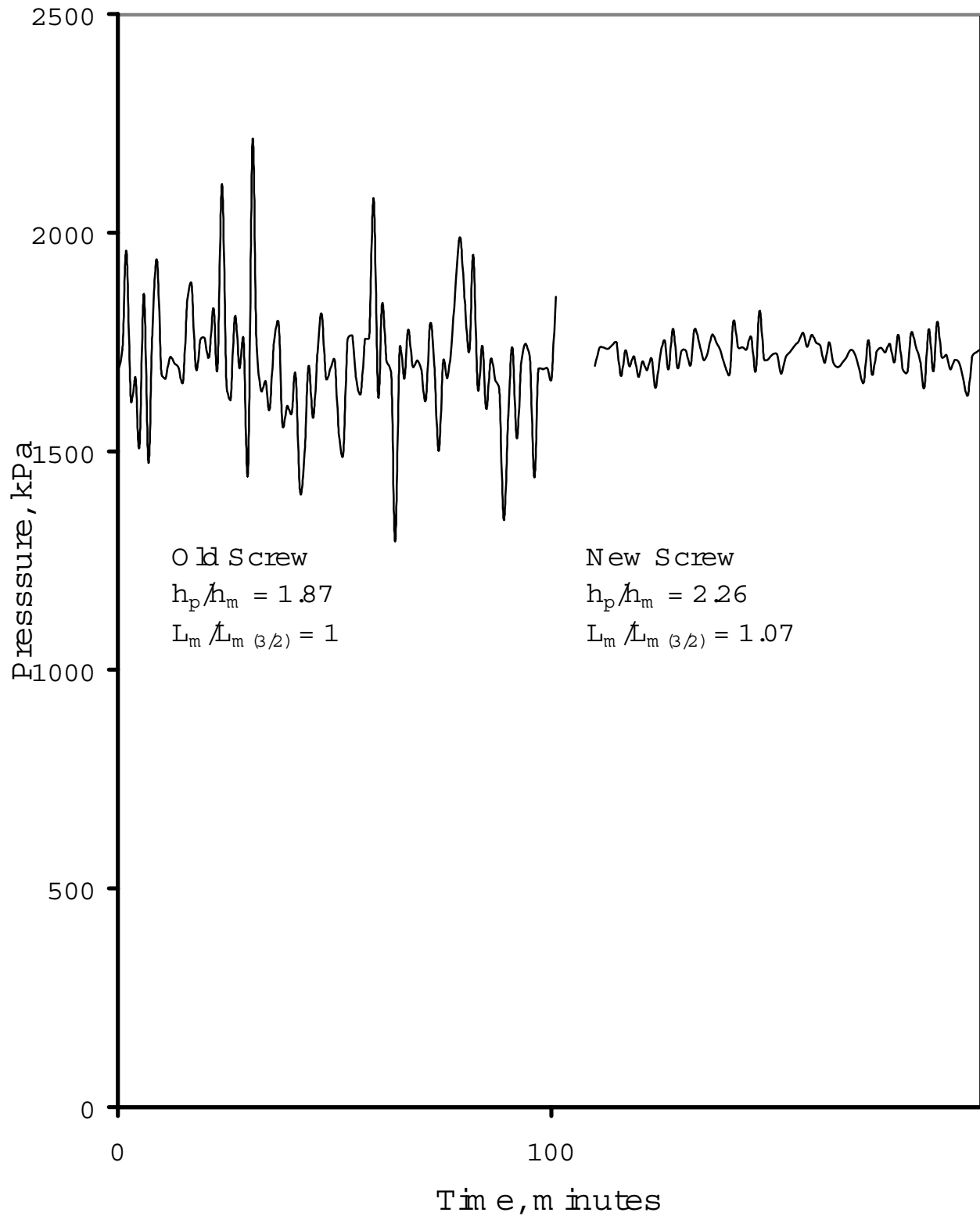


Figure 3. Data for the transient exit pressure for the two screws at identical operating conditions.