KINEMATICS MODEL OF SOLIDS CONVEYING OF LDPE

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Abstract

The kinematics model of solids conveying shows that the solids advance angle is a single empirical factor of solids flow in lieu of friction factors. Furthermore, solids advance angle can be linearly correlated with pressure for LDPE. The linear functions that approximate the solids angle are used with the kinematics model to calculate solids flow of LDPE. The calculations are compared to data to demonstrate the accuracy of the model.

Introduction

Classic extruder screw analysis for pumping of the solids uses a friction factor for the barrel and a second friction factor for the screw as two constitutive relationships that govern the flow. It has recently been shown [1] that the solids advance angle is a single constitutive factor that can be used to describe the performance of solids conveying in extrusion. It was shown that the solids advance angle is a function of pressure for given barrel and screw temperature.

The purpose of this paper is to develop a model for solids flow of LDPE using the solids advance angle function as the constitutive equation. The fundamental kinematics model [1,2] will be combined with the constitutive equation for the solids advance angle to model the solids conveying flow of LDPE.

Solids Advance Angle

The solids advance angle is the angle that the motion of the solids feed plug makes with a line that is *perpendicular* to the axis of the screw as shown in Figure 1. It is bounded by zero degrees at zero flow and 90 degrees at maximum flow. It typically has a value closer to zero degrees.

The solids advance angle, ϕ, (based on cylindrical coordinates) is defined by the kinematics model [1] as

$$\phi = A\tan(v_z / \overline{R}\omega_p), \tag{1}$$

where axial velocity, v_z, is given as

$$v_z = \frac{\dot{m}/\rho_B}{(2\pi\overline{R} - w_f/\sin(\theta))H}, \tag{2}$$

and solids plug rotational speed, ω_p, is given as

$$\omega_p = \omega - \frac{\dot{m}/\rho_B}{\overline{R}Ht_c}\text{,rad/second.} \tag{3}$$

Rotational speed, ω, is given as

$$\omega = 2\pi N, \tag{4}$$

and

$$\overline{R} = R - H/2. \tag{5}$$

The use of cylindrical coordinates in the definition of the solids angle produces a result that is different from the solid advance angle based on Cartesian coordinates [3]. This difference is most pronounced for deep channels, and an example of the difference is given in Figure 2.

Figure 2 shows the calculated solids advance angle in cylindrical coordinates for a 63.5 mm extruder versus a range of typical channel depths, 5 mm to 15 mm. The speed is 50 rpm, and the polymer is LDPE.

The calculated solids advance angle (for the same conditions) in Cartesian coordinates [3] is also shown in Figure 2, and it is significantly different from that for cylindrical coordinates. Also, the difference is shown to be greatest at the largest channel depth of 15 mm. At this channel depth the Cartesian solids advance angle is 25 % lower than the solids advance for cylindrical coordinates.

The geometric screw helix angle, θ, must also be calculated based on the cylindrical coordinate system. It is then given as

$$\theta = a\tan(t/(2\pi\overline{R})). \tag{6}$$

This definition of the helix angle is also a function of channel depth and is shown in Figure 2. It typically becomes larger for deeper channel depths.

Therefore, the results of this work must be used with a solids advance angle and helix angle that are calculated based on cylindrical coordinates as shown. This is especially important for scaling solids conveying to different diameter screws.

Kinematics Model Flow Equation

Flow rate of solids can be calculated based on the kinematics model and the solids advance angle given by Equations 1 to 4. Substitution of Equations 2 through 4 into Equation 1 and solving for flow gives the following result for flow rate in terms of the solids advance angle, ϕ, and the screw helix angle, θ.

$$\dot{m} = \rho_B \omega \overline{R} H t_c \frac{\tan\phi}{\tan\theta + \tan\phi} \tag{7}$$

Equation 7 models the solids flow based on a single constitutive factor, ϕ. Traditionally, two constitutive factors, barrel and screw friction coefficients, are used.

Solids Advance Angle for LDPE

Laboratory data [4] for average flow versus average pressure of LDPE are used to calculate the average solids advance angle. These data are obtained from a test device that isolates the solids conveying portion of single-screw extrusion. Data for flow versus pressure at numerous temperature conditions for barrel and screw are presented. Average values are presented even though the pressure may have had some oscillation.

Equation 7 is solved for solids advance angle, ϕ, to give

$$\phi = a\tan(N_Q \tan(\theta)/(1-N_Q)), \quad (8)$$

where dimensionless flow rate, N_Q, is defined as

$$N_Q = \frac{\dot{m}}{\rho_B \omega \bar{R} H t_c} \text{ , and} \quad (9)$$

where bulk density, ρ_B (kg/cu-m), is also a function of pressure, p (MPa). It is given for LDPE by

$$\rho_B(p) = -2.04p^2 + 47.2p + 593.0. \quad (10)$$

The LDPE flow data [4] are used to calculate the solids advance angle versus pressure with Equations 8, 9 and 10. Solids advance angle versus pressure at the different temperature conditions are then plotted in Figure 3.

Solids Advance Angle Equation

In order to use Equation 7 to calculate the solids flow as a function of pressure, the solids advance angle constitutive equation, $\phi(p)$, is developed that approximates the results of Figure 3. A linear model is suggested [1] .

$$\phi(p) = Ap + B \quad (11\ )$$

The values of A and B depend on barrel and screw temperature, and Table 1 summarizes their values obtained by linear regression of the data of Figure 3.

Table 1

Solids Advance Angle Linear Constants

A, degrees/MPa B, degrees

T_{barrel}	T_{screw}	A	B	R^2
125 °C	50 °C	-3.555	27.90	0.9051
125 °C	100 °C	-3.203	21.73	0.9204
100 °C	50 °C	-1.734	15.96	0.8688
100 °C	100 °C	-1.733	12.53	0.9122
75 °C	50 °C	-0.756	6.145	0.8479
75 °C	100 °C	-0.623	5.605	0.6526
50 °C	50 °C	-0.9025	3.522	0.3053

Data at zero pressure are omitted from the regression, because the assumption of a solids plug used for the kinematics model is likely not valid at zero pressure (loose packed unknown density and partially filled channels.)

The regression coefficient, R^2, is also given in Table 1. For barrel temperatures above 75 °C, it is above 0.84 which indicates that the data are fitted well by the linear assumption. However, the last two conditions show poor correlation, which indicate the assumption that the solids advance angle is only a function of temperatures is not valid for these conditions. A likely cause would be the failure of the solids conveying to behave as a solid plug. This could result from low pressure or from friction internal to the solids bed being lower than that at the barrel or screw surfaces. If this were the case, then slippage internal to the solids bed would occur which would eliminate the unified motion of a solids plug. Slip-stick motion (transients) would also cause scatter in the data for lead angle.

The lack of correlation at some conditions is useful to gain understanding of the process. Since the assumptions of the model are very fundamental (conservation of mass,) the lack of correlation indicates that the process may not be a solid plug, or it may not be steady. The kinematics model with the solids advance angle can isolate those conditions where the process may be fundamentally different. Since solids advance angle is easily calculated from just the flow rate, it makes this approach an excellent tool for trouble shooting and debugging solids conveying problems just by assessing the degree of scatter of the solids advance angle data.

Complete Model

Equation 7 is combined with Equations 10 and 11 to give the complete model of solids flow as a function of pressure for LDPE as

$$\dot{m} = \rho_B(p)\omega \bar{R} H t_c \frac{\tan(Ap+B)}{\tan\theta + \tan(Ap+B)}. \quad (12)$$

This is the basic equation that models the solids conveying flow versus pressure for LDPE. The values of A and B depend on temperature, as shown in Table 1, and the solids conveying flow of LDPE is given as a function of pressure when the values of A and B from Table 1 are used in Equation 12.

The magnitudes of A and B in Table 1 decrease continuously. Therefore, linear interpolation of the values of A and B for temperatures other than those in Table 1 is feasible and would provide reliable results.

The complete model, as given in Equation 12, includes the effect of speed, channel depth, radius, and lead length on the flow rate. It has been shown [1] that the solids lead angle can be assumed to be independent of these parameters, so that the values of A and B can be used for different sized screws and speeds in Equation 12. This makes this model useful for scaling.

Model versus Data

Figures 4-10 plot the results of the model, Equation 12, in comparison with the data [4]. The data contain two values of speed (50 rpm and 80 rpm) and two values of channel depth (8.89 mm and 11.1 mm.) Data for combinations of barrel and screw temperatures the same as in Table 1 are given. Model results for the same barrel and screw temperatures are plotted as lines. The comparison of the data (points) and the model (lines) demonstrates the validity of the assumed linear function of the solids advance lead angle with pressure.

Figures 4-8 are based on data with linear regression coefficients greater than 0.84 for the correlation of A and B (see Table 1) and they show that the model results coincide with the data well. As expected, the model does not predict the flow at zero pressure, as this datum was not used for the linear regression of solids lead angle versus pressure.

The model does predict a slight downward curvature that is typical of the data, but it does not predict the sharp downturn in flow rate at the highest pressures. This sharp downturn in flow rate is most noticeable for the deepest channel. This suggests that there is an effect of channel depth on the solids advance angle that has been neglected, which is most noticeable at the limiting pressure point of the flow rate curve. Also, any difference between data and model over the entire pressure ranges are primarily that the model predicts more flow than the data indicate (Figures 5 and 6 for the 80 rpm and 11.1 mm channel depths.) Again, this difference is most noticeable for the data of the deeper channel. There may be a small affect of channel depth on the relationship of solids angle and pressure, but previous work [1] with screws of a range of diameters and channel depths indicate that effect is slight.

Figures 9 and 10 show the comparison of data and model for solids advance angle at non-ideal temperature conditions. The low regression coefficients (less that 0.66) indicate that other random factors are influencing the solids advance angle. The assumption of a uniform solid plug moving continuously as solids conveying flow is not completely valid for Figures 9 and 10. Factors, such as partially filled channels, slip-stick transients, and slippage within the polymer itself rather that at the walls are possible. However, these factors would be detrimental to any mechanistic model of solids conveying but probably not as readily isolated.

Conclusions

1. Cylindrical coordinates are important to accurate modeling of solids conveying flow.

2. The solids advance angle linearly correlated with pressure for given barrel and screw temperature provides an accurate constitutive equation of solids advance angle for the kinematics model for LDPE.

3. Low flow and low pressure create conditions that violate the assumption of solids plug flow in solids conveying and diminish the accuracy of the model for LDPE. This is evidenced by the poor linear correlation of solids angle with pressure.

Nomenclature

H	channel depth
$\dot{m}$	flow rate
N	screw speed, rpm
N_Q	dimensionless flow rate
p	pressure
R	barrel radius
$\bar{R}$	radius to center of the channel
t	lead length
t_c	channel width in axial direction
T_i	temperature: barrel, i=B, or screw, i=S
v_z	axial resin velocity
w_f	flight width
ϕ	solids lead angle
θ	screw helix angle
ρ_B	bulk density
ω	rotational speed, rad/sec
ω_p	solid plug rotational speed, rad/sec

References

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Key Words

Solids, Conveying, Feed, Model, Advance angle

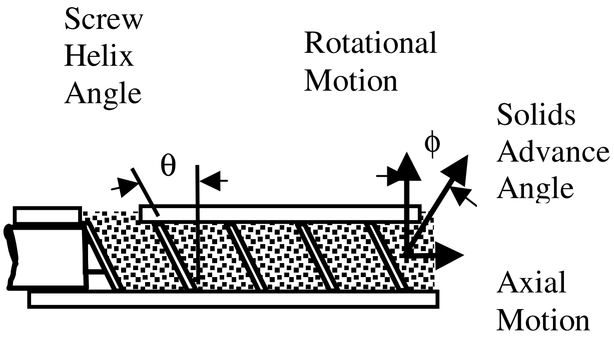

Figure 1. The solids advance angle.

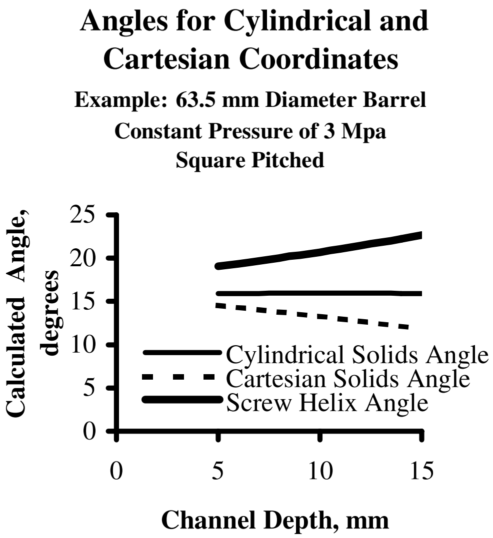

Figure 2. Angles for Cylindrical Coordinates.

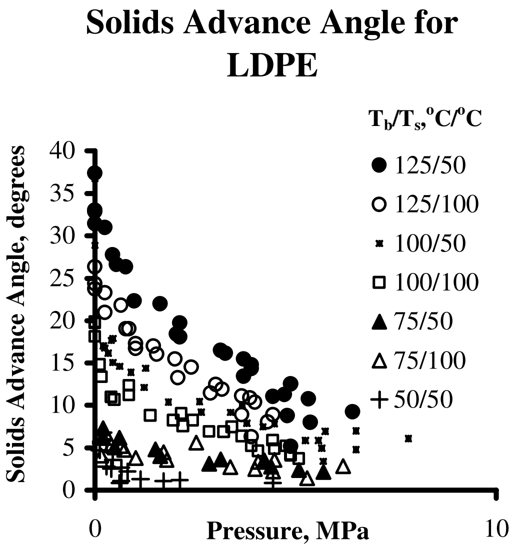

Figure 3. Solids Advance Angles versus pressure for LDPE.

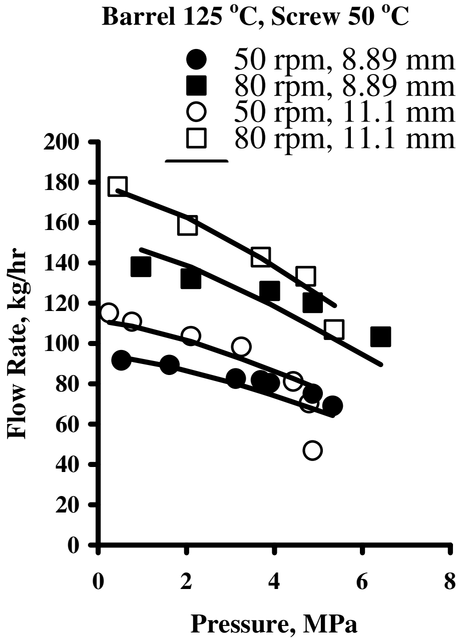

Figure 4. Data and model for 125 °C barrel and 50 °C screw.

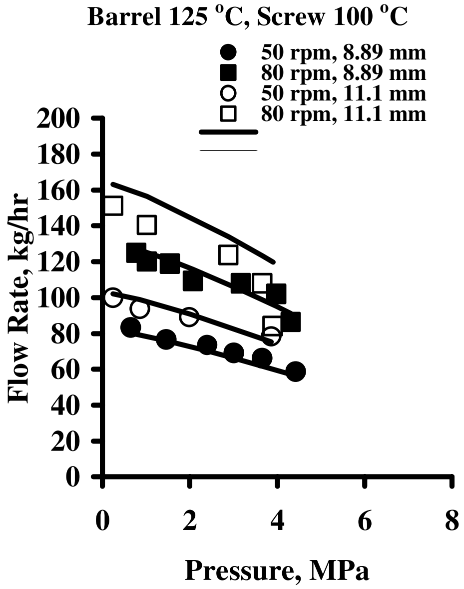

Figure 5. Data and model for 125 °C barrel and 100 °C screw.

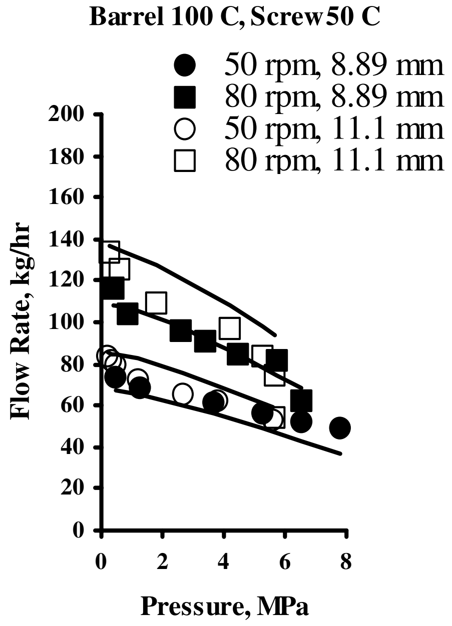

Figure 6. Data and model for 100 °C barrel and 50 °C screw.

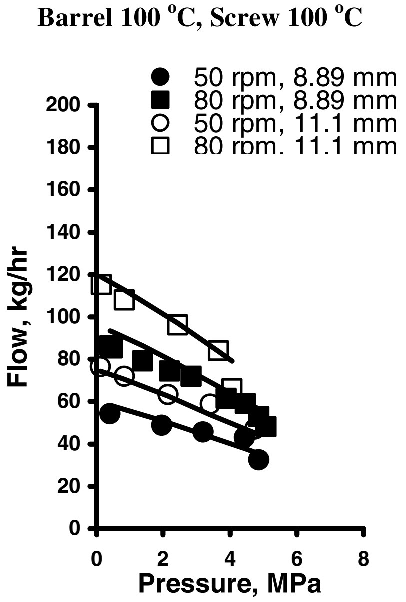

Figure 7. Data and model for 100 °C barrel and 100 °C screw.

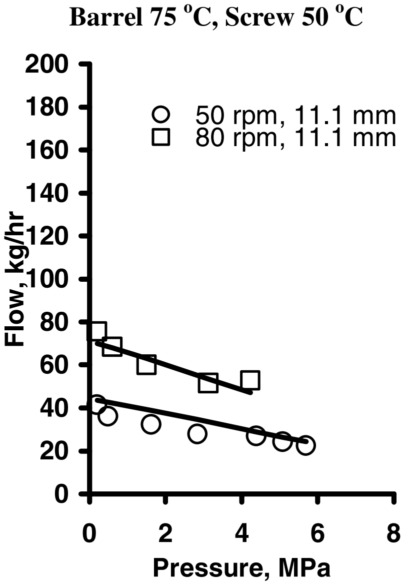

Figure 8. Data and model for 75 °C barrel and 50 °C screw.

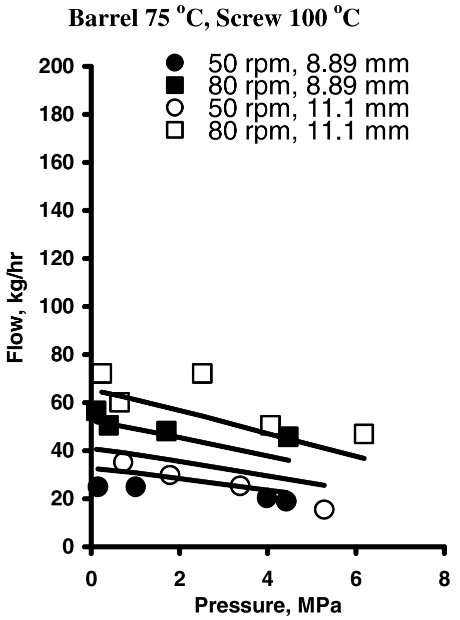

Figure 9. Data and model for 75 °C barrel and 100 °C screw.

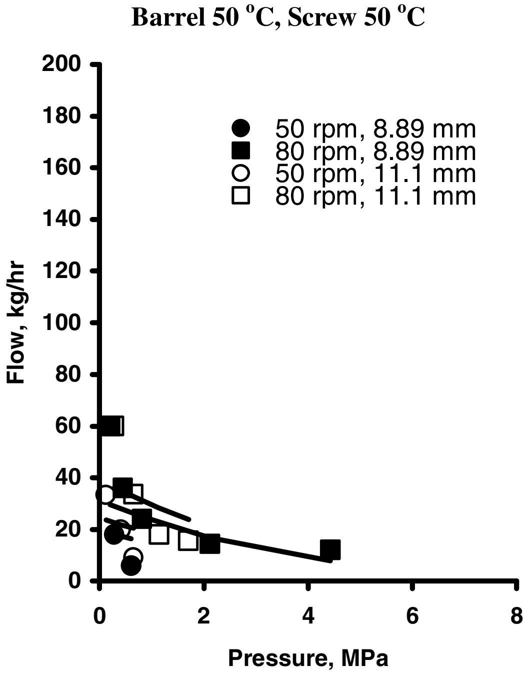

Figure 10. Data and model for 50 °C barrel and 50 °C screw.