Calculating Surge Dampening in Melt Delivery Systems

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Physics of Surge Dampening

The basic physics of dampening is illustrated (Figure 1) by a simplified delivery system. It contains only a delivery pipe and a die. Actual systems contain more elements (filters, mixers, etc.) but this arrangement is sufficient to contain all of the physics that are needed to fully demonstrate the dampening process.

In general, piping is typically chosen in a size to reduce residence time, withstand delivery pressure and temperature, and be economical. All of these factors dictate small diameters and short lengths. Maintaining pressure losses that are within the ability of the extruder dictates larger pipe diameters and short line lengths. However, as will be shown, minimizing the transmission of unwanted flow oscillations requires larger pipes of greater length. Several factors are demonstrated to be important to the transmission of flow oscillations by the delivery system.

The flow that leaves the extruder is assumed to have an oscillating component. This flow oscillation is at known frequency and amplitude. The amplitude of the flow oscillation(at the same frequency) that leaves the die and forms the product is significantly less than that of the extruder as a result of several factors.

Any *flow oscillation* that exits the extruder (enters the delivery system) can (1) compress the polymer, (2) strain the piping system, and (3) be transmitted to the die. The greater the amounts of compression and strain, the smaller the flow oscillation transmitted to the die.

The amounts of compression (1) and strain (2) depend on the *head pulsation* associated with the flow oscillation. The magnitude of head pulsation is determined by frictional fluid losses and inertia of the polymer all along the entire delivery system length.

Polymer Compressibility The bulk compressibility , of the polymer is a key factor in the analysis because it determines the amount of flow that will be absorbed by the compression of the polymer melt in the delivery system. It is given as

$$\beta = \frac{dV/V}{dp}, \quad (1)$$

where V is volume and p is pressure.

Values of can be obtained for polymer melts by differentiating the equation of state [2][3]. The resulting equation given[2] for 1/ is

$$\frac{1}{\beta} = (p + \Pi)[1 + \frac{Mb}{R_G T}(p + \Pi)]$$, (2)

where R_G is the gas constant, 8.314 J/(gmol-K), and T is temperature in Kelvin. Values of the constants M, Π, and b for some polymers are given in the following table [2].

	M, g/mol	Π, mPa	b, m^3/kg
PS	104	186	0.822e-3
Polymethyl methacrylate	100	216	0.734e-3
Ethyl cellulose	60.5	240	0.720e-3
Cellulose acetate butyrate	54.4	285	0.688e-3
PE	28.1	328	0.875e-3

Density The density , of the polymer melt is also needed. It is combined with the compressibility to give the *wave speed* required for the analysis as

$$a_\infty = \sqrt{1/(\beta\rho)}$$. (3)

The wave speed accounts for the effect of compressibility and inertia on the fluctuating portion of the polymer flow.

Piping The wave speed given above is for infinitely rigid piping of the delivery system, and it needs to be modified for the elastic strain of the piping. The piping is assumed to be cylindrical with inside diameter D, the wall thickness e, Young's modulus E, and Poisson's ratio $_p$. The strain of the piping, which absorbs some of the flow oscillation, can be typically described for thick-walled conduits (D/e < 25) as two types: (1) each end of the pipe rigidly fixed (there is no change in the length of the pipe,) or (2) one end fixed and one end free to move. The wave speed is modified[1] to be

$$a = \frac{a_\infty}{\sqrt{1 + (1/(\beta E))(D/e)c_1}}$$, (4)

where

$$c_1 = \frac{2e}{D}(1 + \mu_p) + \frac{D}{(D + e)}(\frac{5}{4} - \mu_p),$$ (5)

if the pipe is fixed at one end only.

If the pipe is rigidly fixed at each end, then

$$c_1 = \frac{2e}{D}(1 + \mu_p) + \frac{D(1 - \mu_p^2)}{D + e} \quad . (6)$$

Frictional Head Loss The melt viscosity , is also needed to give the flow resistance head ΔH_f, for laminar flow Q, in a round pipe of diameter D, and length L, as required by

$$\Delta H_f = [\frac{128\mu}{g\rho\pi D^4}]LQ = [R]LQ \quad , (7)$$

where R is defined as the frictional resistance and g is the gravitational constant.

Impedance Method for Transient Flow

Several mathematical methods[1] are available for modeling transient fluid flow. For steady oscillating flow, as would be found in extrusion lines, the impedance method is convenient. The method is very computationally efficient but requires the use of complex variables. This mathematical complexity is easily performed with a FORTRAN program and a PC.

For the analysis that follows, any parameter that begins with a "Z" is a complex variable. As such, it has two components, a real and imaginary part. However, it is shown in the equations as a single variable, but *all mathematical operations involving "Z" variables are understood to be done with algebra for complex variables.*

The *hydraulic impedance* at any point x, in the delivery system is defined[1] as the complex ratio of the head pulsations Z_{Hx}, to the flow oscillations Z_{Qx}, and it is given as

$$Z(x) = \frac{Z_{Hx}}{Z_{Qx}} \quad . (8)$$

Fundamental to the impedance method is a *propagation constant* Z, for each piping element which is given[1] as

$$Z_{\gamma} = \sqrt{Ag\omega / a^2(-\omega / gA + iR)} \quad , (9)$$

where A is cross-sectional area, ω is frequency, and resistance R, is defined in equation 7.

A *characteristic impedance* Z_C, for each pipe is defined[1] as

$$Z_C = \frac{Z_p a^2}{i\,\omega g A}. \quad (10)$$

The impedance Z, of a point at L relative to a point x, in the delivery system is given [1] as

$$Z(\pm L) = \frac{Z_x - Z_C \tanh(Z_p(\pm L))}{1 - (Z_x / Z_C)\tanh(Z_p(\pm L))}. \quad (11)$$

The sign of L is positive if in the positive mean flow direction from point x, and Z_x is the impedance at point x.

A special case for impedance exists for an element of the delivery system which is very short and is assumed to have no fluid inertia or capacity (little mass or volume) but have significant frictional pressure loss. Dies and filter elements are assumed to have only frictional loss, and the impedance of such purely resistive elements in equation 11 yields

$$Z(\pm L) = Z_x \pm H / Q, \quad (12)$$

where the head H, and volume flow Q, are obtained from steady-state measurements or calculations.

The head at a point L, distance from another point x, is given[1] as

$$Z_{HL} = Z_{Hx}\cosh(\pm L Z_p) - Z_{Qx} Z_C \sinh(\pm L Z_p).$$

(13)

Similarly, flow at a point L, distance from another point x, is given[1] as

$$Z_{QL} = -\frac{Z_{Hx}}{Z_C}\sinh(\pm L Z_p) + Z_{Qx}\cosh(\pm L Z_p). \quad (14)$$

Two boundary conditions of any combination of flow or pressure at either end of the delivery system are needed to solve the equations. Normally, the boundary condition at the exit of the delivery system is zero impedance since the pressure is steady. The other boundary condition at the inlet to the delivery system is usually a driving function, either flow or pressure as a function of time.

Equations 8-14 and the boundary conditions are then used to find the head and flow surges along the delivery system. The calculations are done in complex variables, which is easily accomplished with FORTRAN or other scientific software that supports complex variables. The sequence of the algorithm is shown in Figure 2.

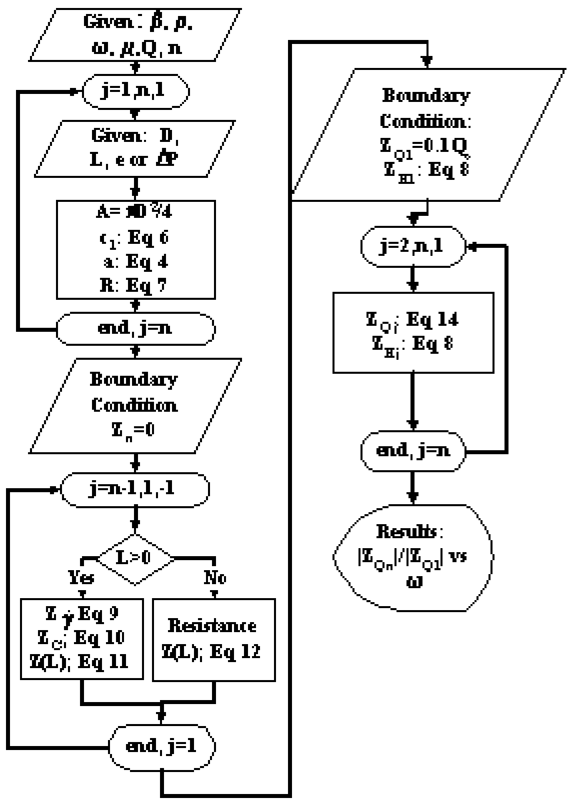

Figure 2. Flow Chart for Impedance Program.

The number of junctions in the piping is a variable n, as shown in Figure 2. This variable permits any number of elements to be connected (in series) to simulate an arrangement of many more elements. The n junctions are numbered in consecutive order as are the (n-1) connecting elements (pipes, dies, filters, etc.)

Example

A numerical example of Figure 1 is now illustrated. Assume the extruder is 63 mm diameter operating at 100 kg/hr of PE at 250 ^{o}C(533 K) with a 10 kg/hr flow oscillation (10% of average flow.) The pipe length is one-meter and the inside diameter is 18.8 mm. The steel pipe wall is 3.9 mm thickness. There are 3 junctions and 2 elements.

The pipe is connected to the die with a known mean pressure loss of 1.38 mPa at 100 kg/hr. The pipe is assumed to be rigidly attached at both ends.

Properties of PE are shown in the table. The melt density is assumed to be 730 kg/m^3, and the melt viscosity is assumed to be 90 Pa-s at 250 ^{o}C.

Equation 2 is first used to calculate the compressibility of the PE using the constants given for it in the table. The result is

$$\frac{1}{\beta} = (2.1 + 328)[1 + \frac{(28.1)(0.875)}{(8.314)(533)}(2.1 + 328)]$$
$$= 935\ mPa. \qquad (15)$$

Assuming a density of 730 kg/m^3, the volume flow rate is given by

$$Q = 100 / 730 / 3600 = 3.8e - 5m^3 / s. \qquad (16)$$

Results

A FORTRAN program, which is flow charted according to Figure 2, is used to calculate the flow fluctuations of the example. The results are plotted as the ratio of flow oscillation at the exit to that at the extruder as a function of frequency. Figure 3 shows results for the original piping and different diameters of piping, Figure 4 is for different resistances of the die, and Figure 5 is for different wave speeds, which represents the effect a change in polymer will have on the product uniformity. Finally, Figure 6 is the effect of longer piping between extruder and die.

The basic case of Figure 3 illustrates the importance of piping diameter on the level of dampening by the system. Four IDs are used, ranging from 18.8 mm to 32.3 mm. This range of diameters is logical for consideration of flow of 100 kg/hr. As illustrated, a significant increase in dampening occurs as the diameter is increased. At the highest frequency (1200 cpm) the dampening of the flow oscillations is about twice for the 32.3mm ID over the 18.8 mm ID. At lower frequencies, (e.g. less than 200 cpm) the greater dampening of the larger piping is not as significant. However, at 200 cpm the dampening of the larger pipe is about 0.7 and that of the original smaller pipe is only 0.95. This is about a 25% improvement in uniformity.

Of interest is the steady pressure loss that the one- meter pipe produces. For the 18.8 mm pipe at 100 kg/hr and 90 Pa-s viscosity, the pressure loss is about 67 kPa (Hagen-Poiseuille equation), which is less than 5% of the pressure of the die (1380 kPa.) Therefore, it adds very little to the pumping load of the die, and it is an acceptable diameter from that standpoint.

However, by increasing the diameter to 32.3 mm, the dampening of the oscillations is about 25% less than for the diameter of 18.8 mm. The pressure loss is calculated to be about 7.7 kPa for this diameter pipe, which seems too conservative from the pumping load requirement. In the interest of product uniformity, the choice of the larger piping would clearly lower the influence of extruder surges even though not necessary for steady pressure loss considerations.

Figure 4 illustrates the effect of using a die with a larger pressure loss. The pressure loss is increased by 50% to 2.07 mPa for 100 kg/hr. Again, four diameters are shown, and the amount of dampening is seen to be increased over that of Figure 3 at 1.38 mPa die pressure for all cases and frequencies. However, the improvement is not as great as that for the nominal increase in diameter demonstrated by Figure 2.

Figure 5 demonstrates the effect of wave speed on the dampening. The wave speed, as discussed earlier, is a function of the polymer and the piping. Anything that can "soften" the system will lower the wave speed, namely a more compressible polymer (larger β) or more flexible piping. Trapped gases or air will also serve to soften the system, which can be troublesome as this is likely to occur randomly.

Figure 6 illustrates the influence of pipe length on the dampening of the oscillations. The pipe length has been increased from 1 m to 2 m, and the result is increased dampening to about twice the amount at all frequencies as for the shorter length of Figure 3. The length of pipe is normally kept to a minimum in the interest of residence time, cost, and space conservation. However, improved flow dampening will require some minimum length of delivery piping.

Conclusion

The delivery system for a polymer melt can have a significant effect on the amount of flow oscillation produced by the extruder that reaches the die exit and the product. The relative amount of flow dampening caused by pipe diameter and length, pressure, frequency, and polymer can be calculated with a computer program.

Recommendation

The program, as demonstrated here, can be applied to much more complicated and complete systems. It is relatively easy to program from the flow chart and equations if a language that provides complex variables (e.g., FORTRAN) is used. Once programmed, combinations of size and configuration of the delivery system can be judged for their relative dampening ability, and dampening can be included in the strategy for delivery system design along with pressure drop, residence time, and economics.

References

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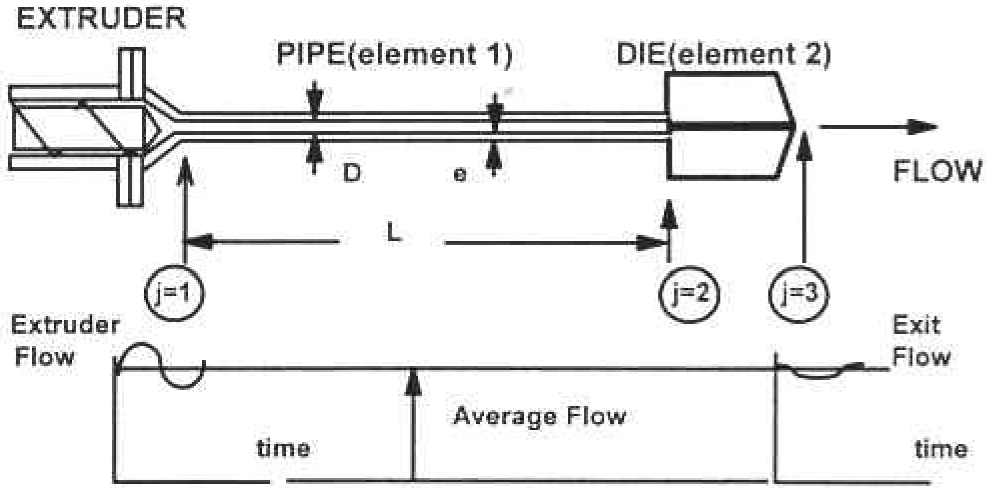

Figure 1. The single pipe and die to demonstrate the impedance method for transient flow dampening.

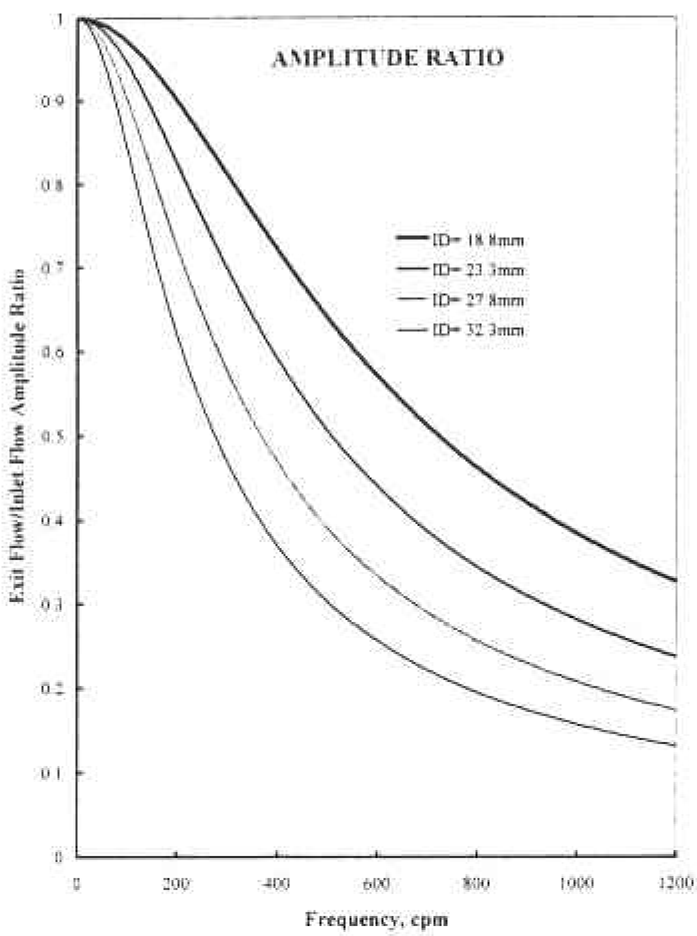

Figure 3. Results for the example. Pipe length is 1 meter, die pressure drop is 1.38 mPa, and wave speed is 1115 m/s. Results for four inside diameters are shown.

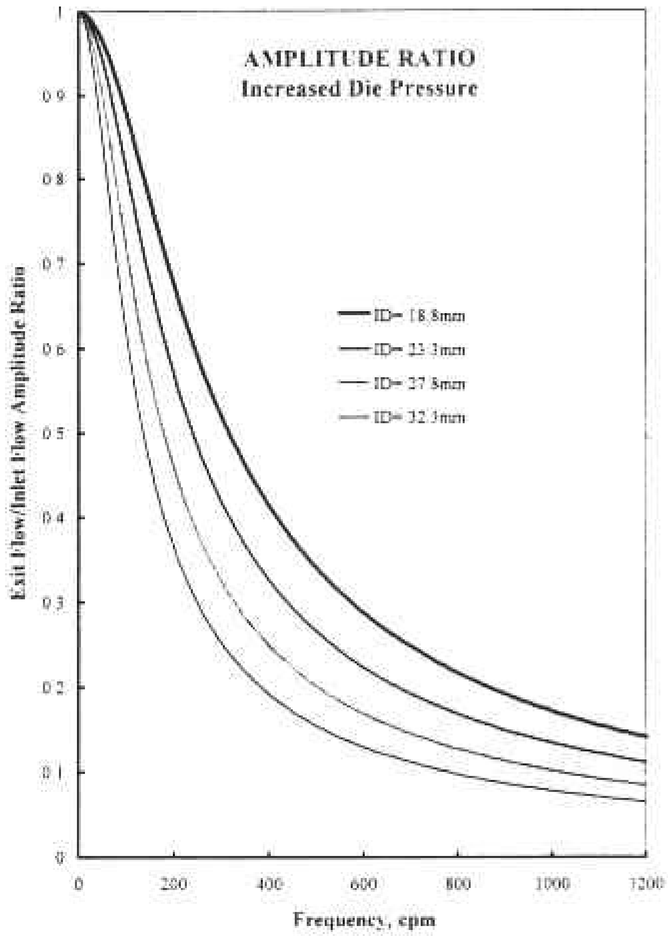

Figure 4. Results for the example with the die pressure increased to 2.07 mPa. Pipe length is 1 meter, and wave speed is 1115 m/s.

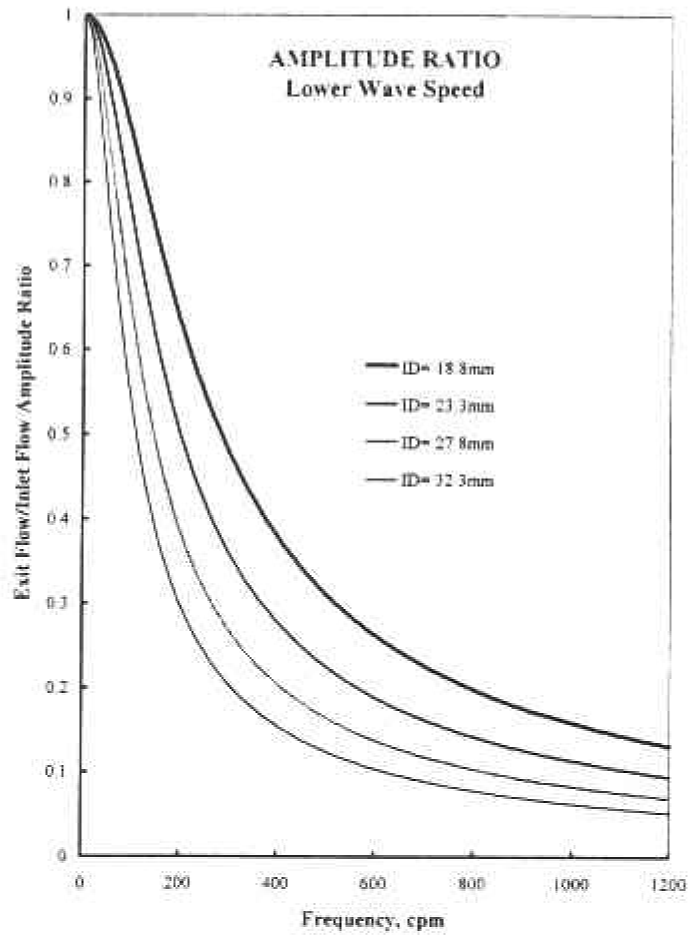

Figure 5 Results for the example with lower wave speed. Pipe length is 1 meter, die pressure drop is 1.38 mPa, and wave speed is 700 m/s

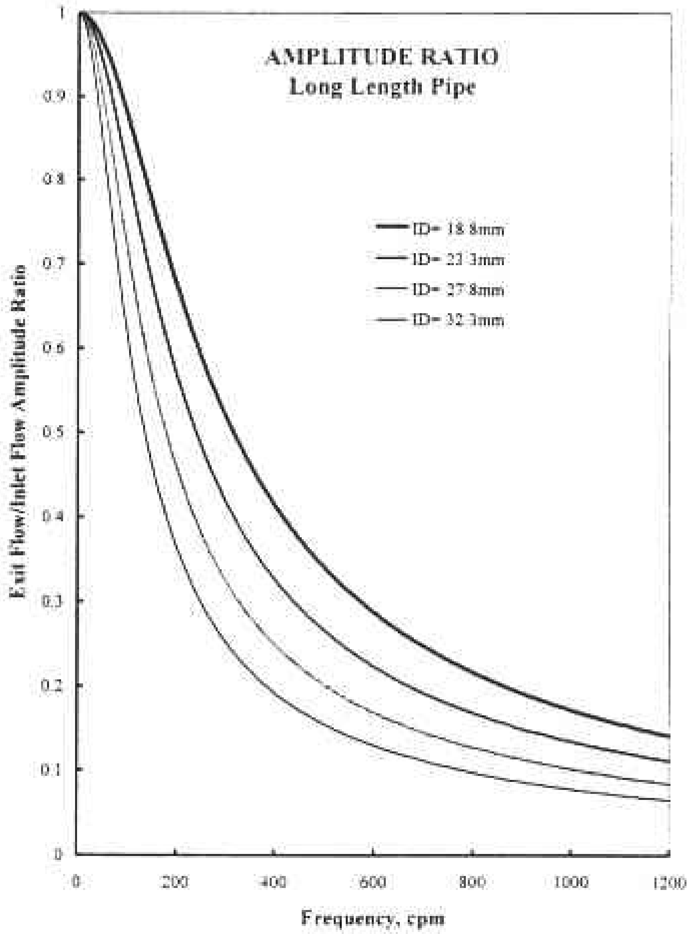

Figure 6 Pipe length is **2 meters**, die pressure drop is 1.38 mPa, and wave speed is 1115 m/s. Results for four inside diameters are shown.

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